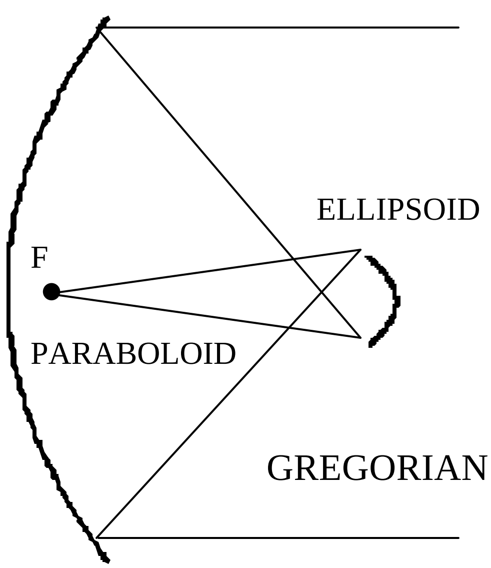
Antennas & Propagation

LECTURE NOTES
VOLUME IV

APERTURES, HORNS AND REFLECTORS

by Professor David Jenn



(*ver1.3)*

Equivalence Principle (1)

There is symmetry between the electric and magnetic quantities that occur in electro-magnetics. This relationship is referred to as <u>duality</u>. However, a major difference between the two views is that there are no magnetic charges and therefore no magnetic current. Fictitious magnetic current $\vec{J}_m$ and charge r_{v_m} can be introduced

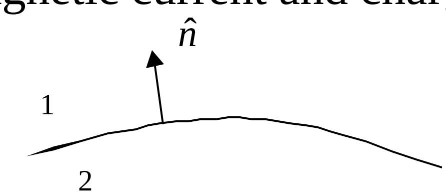
$$(1)\nabla \times \vec{E} = -jwm\vec{H} - \vec{J}_m \qquad (3)\nabla \cdot \vec{H} = r_{vm} / m$$
$$(2)\nabla \times \vec{H} = \vec{J} + jwe\vec{E} \qquad (4)\nabla \cdot \vec{E} = r_v / e$$

If magnetic current is allowed, then the radiation integrals must be modified. The far field radiation integral becomes

$$\vec{E}(\vec{r}) \approx \frac{-jkh}{4pr} e^{-jkr} \iint_S \left[\vec{J}_s - \hat{r}\left(\vec{J}_s \bullet \hat{r}\right) + \frac{1}{h} \vec{J}_{ms} \times \hat{r} \right] e^{jk(\vec{r}' \bullet \hat{r})} ds'$$

where $\vec{J}_{ms}$ is the magnetic surface current density (V/m). The boundary conditions at an interface must also be modified to include the magnetic current and charg

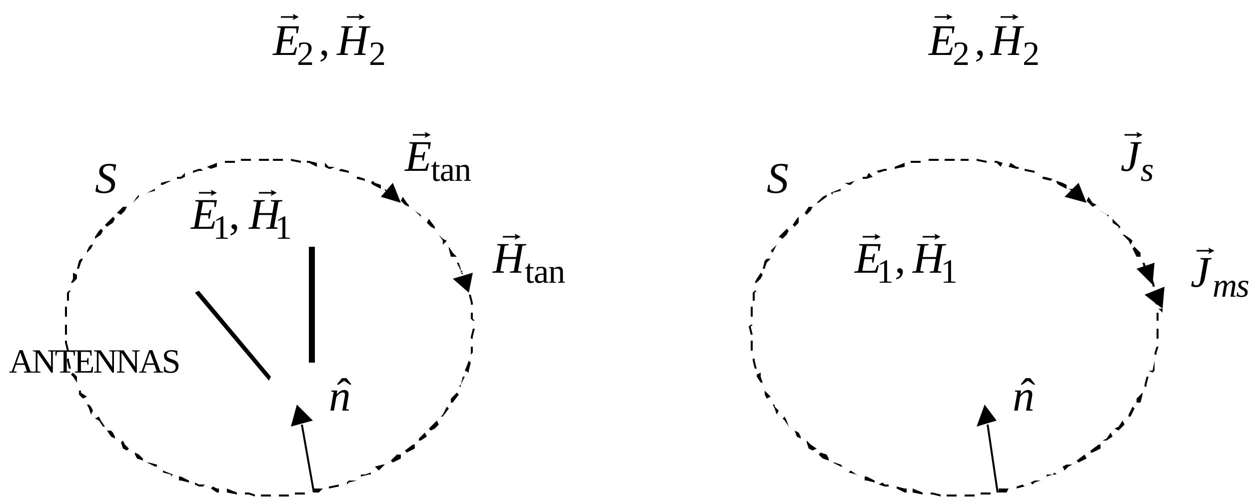
$$-\hat{n} \times \left(\vec{E}_1 - \vec{E}_2\right) = \vec{J}_{ms}$$
$$\nabla \cdot \left(\vec{H}_1 - \vec{H}_2\right) = r_{vs} / m$$



Equivalence Principle (2)

In some cases it will be advantageous to replace an actual current distribution with an equivalent one over a simpler surface. An example is illustrated below. The currents on the antennas inside of an arbitrary surface *S* set up electric and magnetic fields everywhere. The same external fields will exist if the antennas are removed and replaced with the proper equivalent currents on the surface.

ORIGINAL PROBLEM EQUIVALENT PROBLEM



The required surface currents are:

$$\vec{J}_{ms} = -\hat{n} \times \left(\vec{E}_1 - \vec{E}_2\right) \quad \text{and} \quad \vec{J}_s = \hat{n} \times \left(\vec{H}_1 - \vec{H}_2\right)$$

Equivalence Principle (3)

Important points regarding the equivalence principle:

1. The tangential fields are sufficient to completely define the fields everywhere in space, both inside and outside of *S*.

2. If the fields inside do not have to be identical to those in the original problem, then the currents to provide the same external fields are not unique.

3. Love's equivalence principle refers to the case where the interior fields are set to zero. The equivalent currents become

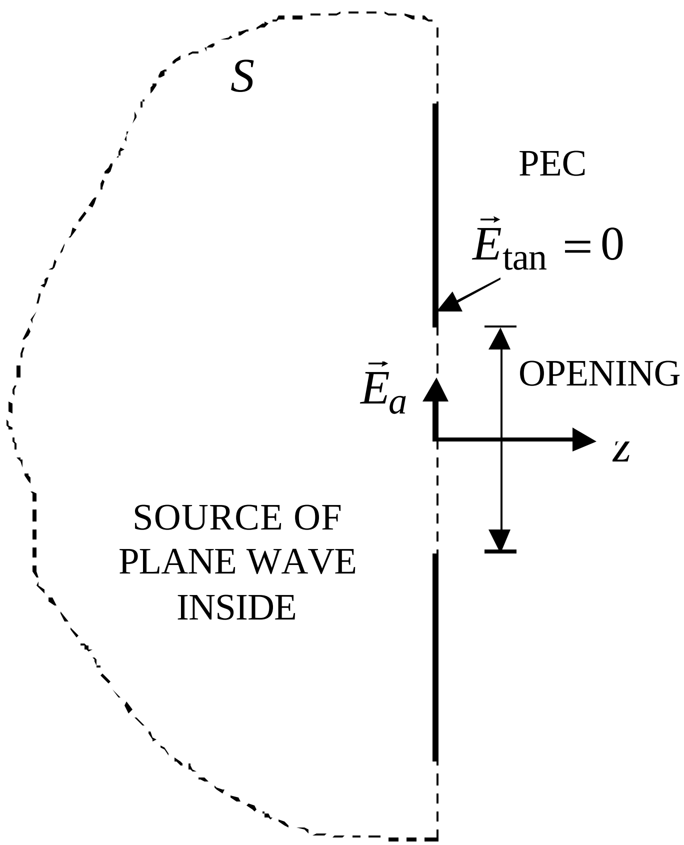
$$\vec{J}_{ms} = \hat{n} \times \vec{E}_2$$
$$\vec{J}_s = -\hat{n} \times \vec{H}_2$$

or in terms of the outward normal

$$\vec{J}_{ms} = -\hat{n} \times \vec{E}_2$$
$$\vec{J}_s = \hat{n} \times \vec{H}_2$$

Apertures (1)

The equivalence principle can be used to determine the radiation from an aperture (opening) in an infinite ground plane. The aperture lies in the $z = 0$ plane. Region 1 contains the source.



In order to apply the radiation integrals, we need to find the currents in unbounded space (no objects present).

- Apply Love's equivalence principle to find the currents on *S*. The currents are nonzero only in the aperture.

- Both electric and magnetic currents exist in the aperture. To simplify the integration we would like to eliminate one of the currents. Since $\vec{E}_a$ is specified, we will use the magnetic current.

The steps involved in eliminating the electric current are illustrated in the figure on the next page.

Apertures (2)

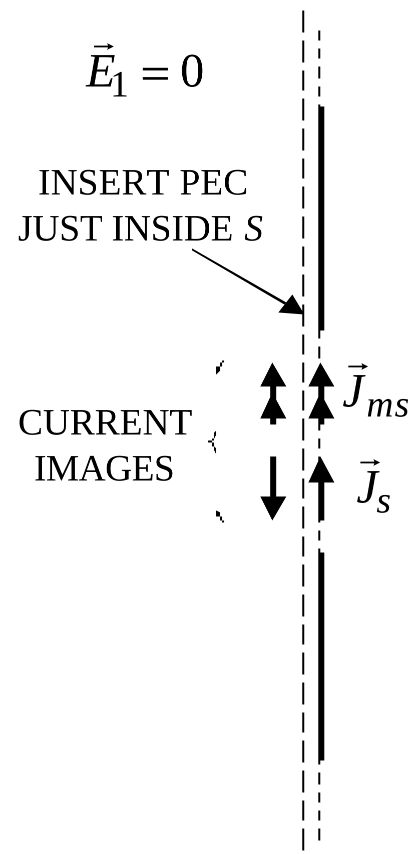
1. Since $\vec{E}_1 = \vec{H}_1 = 0$ inside, we can place any object inside without affecting the fields. Put a PEC just inside of region 1.

2. Now remove the PEC and introduce images of the sources $\vec{J}_s$ and $\vec{J}_{ms}$

3. Allow the images and sources to approach the PEC. The PEC shorts out the electric current. (The image of an electric current element is opposite the source.) Only the magnetic current remains.

$$\vec{J}_{ms} = \begin{cases} -2\hat{n}\times\vec{E}_2 = -2\hat{n}\times\vec{E}_a, & \text{in the aperture} \\ 0, & \text{else} \end{cases}$$

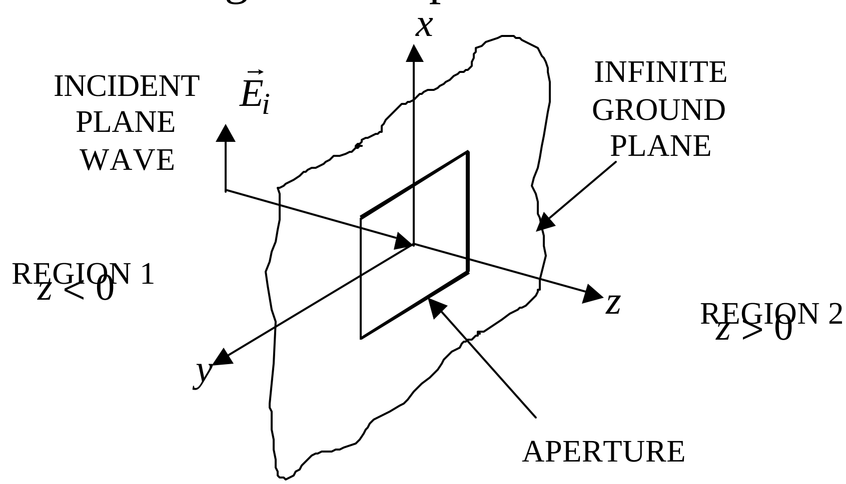
Note: Alternatively a perfect magnetic conductor (PMC) could be placed inside *S*. The magnetic current would short out and the electric current would double.



Rectangular Aperture (1)

One basic application of the equivalence principle is radiation from a rectangular aperture of width 2b (in y) and height 2a (in x). Assume that the incident plane wave is $\vec{E}_i = \hat{x} E_o e^{-jkz}$. Evaluating the incident field at z = 0 gives the aperture field

$$\vec{E}_a = \begin{cases} \hat{x} E_o, & |x| \le a, |y| \le b \\ 0, & \text{else} \end{cases}$$



The equivalent current in the aperture is

$$\vec{J}_{ms} = -2\hat{n} \times \vec{E}_a = -2 E_o \hat{y}$$

All objects are removed so that the currents exists alone in free space.

Now the radiation integral can be applied. Since the electric current is zero, the far field at observation points in region 2 is

$$\vec{E}(\vec{r}) = \frac{-jk}{4\pi r} e^{-jkr} \iint_S \vec{J}_{ms} \times \hat{r} e^{jk(\vec{r}\,' \bullet \hat{r})} ds'$$

where $\vec{J}_{ms} \times \hat{r} = -2 E_o \hat{y} \times (\hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta)$

$$= 2 E_o (\hat{z} \sin\theta \cos\phi - \hat{x} \cos\theta)$$

Rectangular Aperture (2)

The position vector to an integration point in the aperture is $\vec{r}\,' = \hat{x}x' + \hat{y}y'$ and therefore the dot product in the exponent is

$$\hat{r} \bullet \vec{r}\,' = x'\sin\theta\cos\phi + y'\sin\theta\sin\phi$$

The integral becomes

$$\vec{E}(\vec{r}) = \frac{-jk}{4\pi r}e^{-jkr}\,2E_o\left(\hat{z}\sin\theta\cos\phi - \hat{x}\cos\theta\right)\underbrace{\int_{-a}^{a} e^{jkx'\sin\theta\cos\phi}\,dx'}_{2a\,\mathrm{sinc}(ka\sin\theta\cos\phi)}\;\underbrace{\int_{-b}^{b} e^{jky'\sin\theta\sin\phi}\,dy'}_{2b\,\mathrm{sinc}(kb\sin\theta\sin\phi)}$$

The dot products with the spherical components, $\hat{z}\bullet\hat{\theta} = -\sin\theta$ and $\hat{x}\bullet\hat{\theta} = \cos\theta\cos\phi$ lead to

$$\hat{\theta}\bullet\left(\hat{z}\sin\theta\cos\phi - \hat{x}\cos\theta\right) = -\sin^2\theta\cos\phi - \cos^2\theta = \cos\phi$$

Using the fact that the aperture area is $A = 4ab$ gives

$$E_\theta = \frac{jkAE_o}{2\pi r}e^{-jkr}\cos\phi\,\mathrm{sinc}(ka\sin\theta\cos\phi)\,\mathrm{sinc}(kb\sin\theta\sin\phi)$$

where r is the distance from the center of the aperture to the observation point.

Rectangular Aperture (3)

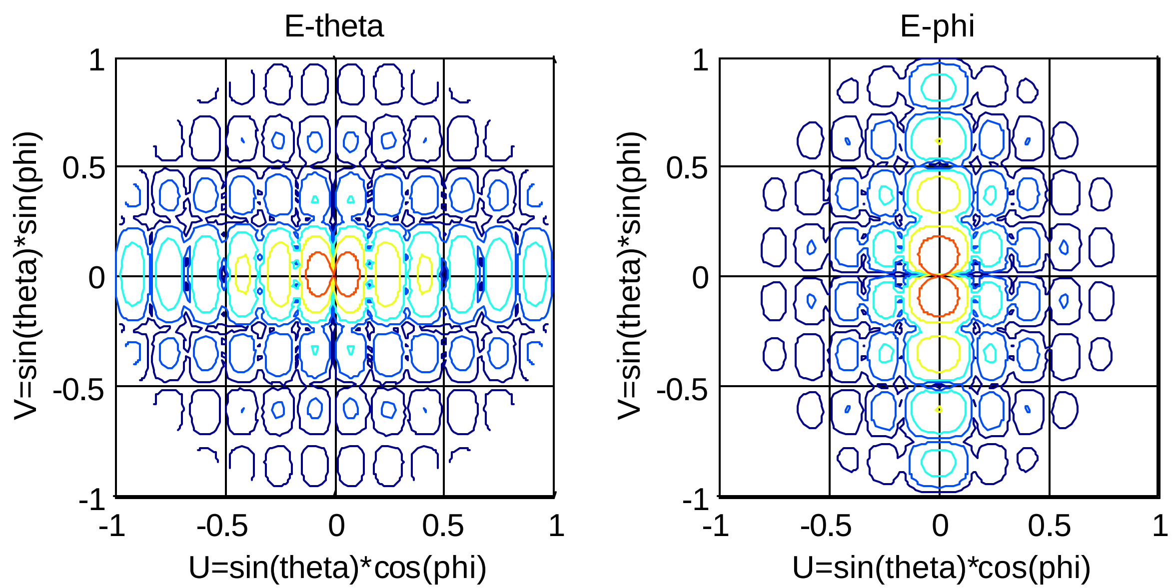
Similarly, the dot products $\hat{z} \bullet \hat{f} = 0$ and $\hat{x} \bullet \hat{f} = -\sin f$ lead to

$$\hat{f} \bullet (\hat{z} \sin q \cos f - \hat{x} \cos q) = \sin f \cos q$$

and

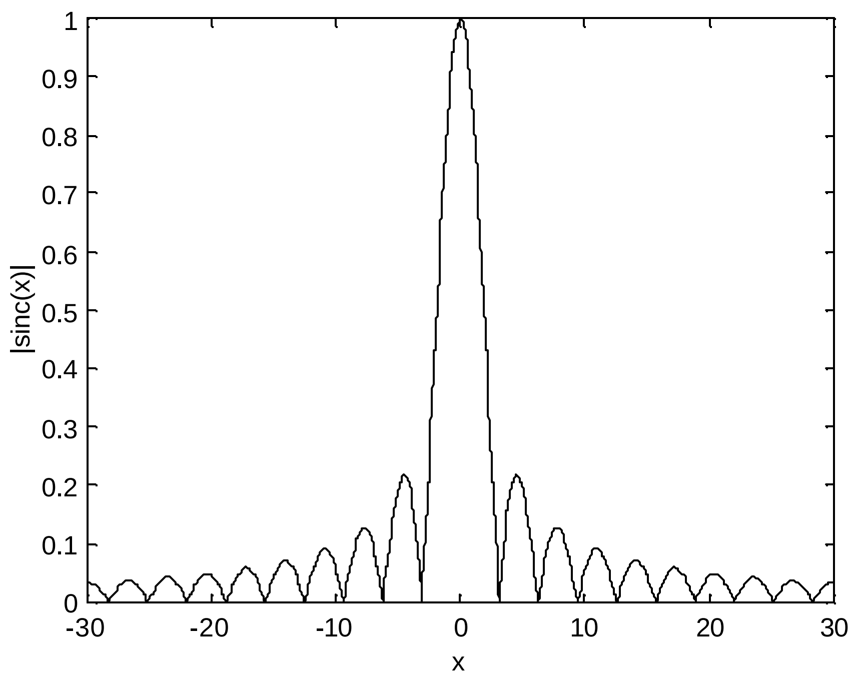
$$E_f = \frac{-jkAE_o}{2pr} e^{-jkr} \cos q \sin f \; \mathrm{sinc}(ka \sin q \cos f)\, \mathrm{sinc}(kb \sin q \sin f)$$

Example: Contour plots for $a = 3l$ and $b = 2l$ in direction cosines are shown



Rectangular Aperture (4)

Properties of the "sinc" function



- Maximum value at x =0

$$\sin(0) = \left.\frac{\sin(x)}{x}\right|_{x=0} = 1$$

- First sidelobe level: -13.2 dB below the maximum
- Caution: some authors and Matlab define

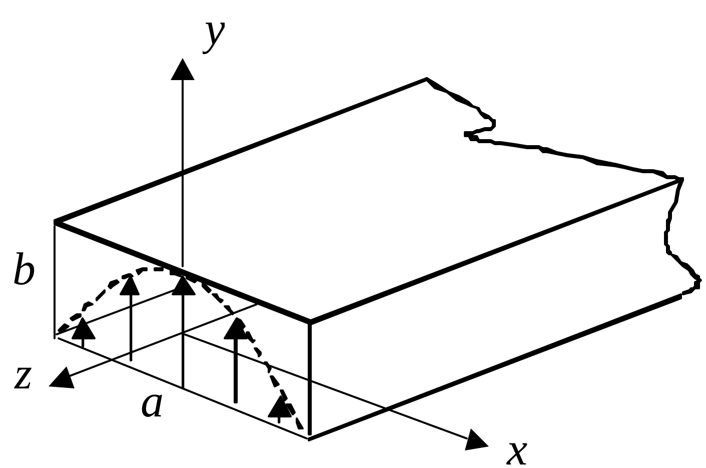
$$\sin(x) = \frac{\sin(px)}{x}$$

Tapered Aperture (1)

Just as in the case of array antennas, the sidelobe level can be reduced and the main beam scanned by controlling the amplitude and phase of the aperture field. As an example, let a rectangular aperture be excited by the TE_{10} mode from a waveguide. The field in the aperture is given by

$$\vec{E}_a = \begin{cases} \hat{y} E_o \cos(\pi x'/a), & |x'| \le a/2 \text{ and } |y'| \le b/2 \\ 0, & \text{else} \end{cases}$$

The equivalent magnetic current is $\vec{J}_{ms} = -2\hat{n} \times \vec{E}_a = -2(\overbrace{\hat{z} \times \hat{y}}^{=-\hat{x}})E_o \cos(\pi x'/a)$ in the aperture.



Tapered Aperture (2)

The radiation integral is

$$\vec{E}(\vec{r}) = \frac{-jkE_o}{2\pi r} e^{-jkr} [\hat{x} \times \hat{r}] \iint_S \cos(\pi x'/a) e^{jk(\vec{r}' \bullet \hat{r})} dx' dy'$$

The cross product reduces to

$$\hat{x} \times \hat{r} = \hat{z} \sin\theta \sin\phi - \hat{y} \cos\theta$$

The integrals are separable. The y integral is the same as the uniformly illuminated case

$$\vec{E}(\vec{r}) = \frac{-jk}{2\pi r} e^{-jkr} E_o (\hat{z} \sin\theta \sin\phi - \hat{y} \cos\theta)$$
$$\times \underbrace{\int_{-a/2}^{a/2} \cos(\pi x'/a) e^{jkx' \sin\theta \cos\phi} dx'}_{\frac{2\pi a \cos\left(\frac{ka}{2} \sin\theta \cos\phi\right)}{\pi^2 - (ka \sin\theta \cos\phi)^2}} \underbrace{\int_{-b/2}^{b/2} e^{jky' \sin\theta \sin\phi} dy'}_{b \operatorname{sinc}\left(\frac{kb}{2} \sin\theta \sin\phi\right)}$$

Tapered Aperture (3)

The q component is obtained from

$$\hat{q} \bullet (\hat{z}\sin q \sin f - \hat{y}\cos q) = -\sin^2 q \sin f - \cos^2 q \sin f = -\sin f$$

or,

$$E_q = \frac{jkE_o A}{r} e^{-jkr} \sin f \left(\frac{\cos\left(\frac{ka}{2}\sin q \cos f\right)}{p^2 - (ka \sin q \cos f)^2} \right) \left(\mathrm{sinc}\left(\frac{kb}{2}\sin q \sin f\right) \right)$$

The aperture illumination efficiency is

$$e_i = \frac{\left| \iint\limits_S \hat{n} \times \vec{E}_a \, dxdy \right|^2}{A \iint\limits_S \left| \hat{n} \times \vec{E}_a \right|^2 dxdy}$$

The numerator is

$$\left| \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \hat{z} \times \hat{y} E_o \cos(px'/a) dx'dy' \right| = bE_o \left[\frac{a}{p} \sin(px'/a) \right]_{-a/2}^{a/2} = \frac{2abE_o}{p}$$

Tapered Aperture (4)

The denominator is

$$\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\left|\hat{z}\times\hat{y}E_o\cos(px'/a)\right|^2 dx'dy' = bE_o^2\int_{-a/2}^{a/2}\cos^2(px'/a)dx' = \frac{abE_o^2}{2}$$

The ratio gives the <u>illumination</u> (or <u>taper</u>) <u>efficiency</u>,

$$e_i = \frac{\left|2abE_o/p\right|^2}{AabE_o^2/2} = 8/p^2$$

The directivity is

$$D = \frac{4pA}{l^2}e_i = \frac{32A}{l^2 p}\underbrace{\left(\frac{2p/l}{k}\right)}_{=1} = \left(\frac{64}{kl}\right)\left(\frac{A}{l^2}\right)$$

Example: WR-90 waveguide (a = 0.9 inch, b = 0.4 inch) and l = 3 cm: D = 2.63.

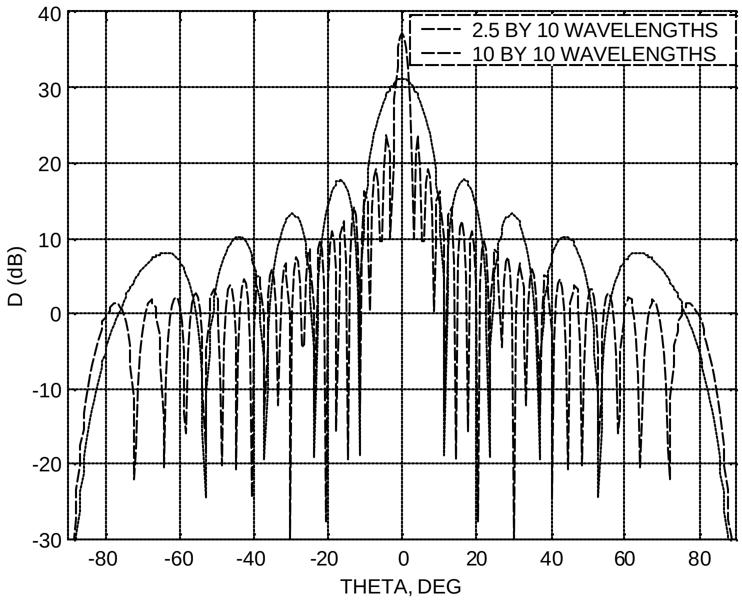
Summary of Aperture Distributions

This table is similar to Table 7.1 from Skolnik presented previously. This table includes entries for circular apertures. (Note: x and r are normalized aperture variables and $a(x) = |A(x)|$, where $A(x)$ is the complex illumination coefficient.

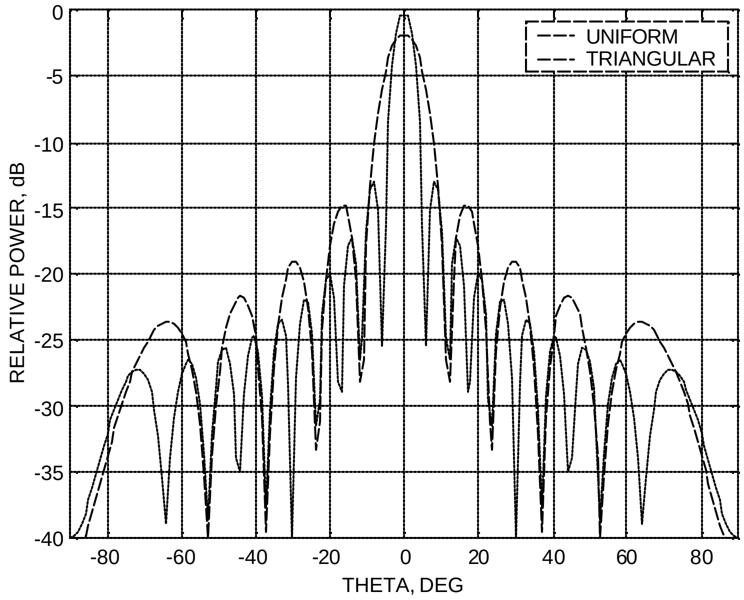
FIRST SIDELOBE LEVEL, DB	3 DB BEAM-WIDTH, RADIANS	LINEAR APERTURE		CIRCULAR APERTURE	
		$a(x)$	e_i	$a(r)$	e_i
13.2	$0.88 l/(2a)$	1	1	1	1
17.6	$1.02 l/(2a)$	$\sqrt{1-x^2}$	0.865	$\sqrt{1-r^2}$	0.75
20.6	$1.15 l/(2a)$	$1-x^2$	0.833	$1-r^2$	0.64
24.6	$1.27 l/(2a)$	$(1-x^2)^{3/2}$	0.75	$(1-r^2)^{3/2}$	
28.6	$1.36 l/(2a)$	$(1-x^2)^2$	0.68	$(1-r^2)^2$	0.55
30.6	$1.47 l/(2a)$	$(1-x^2)^{5/2}$	0.652		

Radiation Patterns From Apertures

Comparison of patterns for different aperture widths

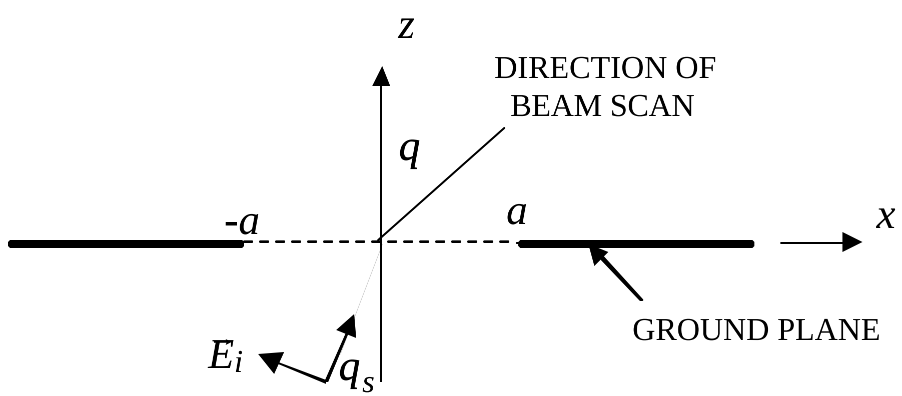


Uniform vs. triangular aperture illumination



Scanned Aperture

A linear phase progression across the aperture causes the beam to scan.



The far field has the same form as the non-scanned case, but with the argument modified to include the linear phase

$$\vec{E} = \frac{jkAE_o}{2pr} e^{-jkr} \left(\hat{q} \cos f - \hat{f} \sin f \cos q \right)$$
$$\times \mathrm{sinc}\left[ka(\sin q \cos f - \sin q_s \cos f_s) \right] \mathrm{sinc}\left[kb(\sin q \sin f - \sin q_s \sin f_s) \right]$$

Example: What phase shift is required to scan the beam of an aperture with $2a = 10\,l$ to 30°?

$$k(2a)\sin 30^\circ \cos 0^\circ = \frac{2p(10\,l)}{l}(0.5) = 10p = 1800^\circ$$

Aperture Example

Example: A radar antenna requires a beamwidth of 25 degrees in elevation and 2 degrees in azimuth. The azimuth sidelobes must be 30 dB and the elevation sidelobes 20 dB. Find *a*, *b* and *G*.

Let the *x*-*z* plane be azimuth and the *y*-*z* plane elevation. Based on the required sidelobe levels, from the table,

Azimuth HPBW: $(2^\circ)\left(\frac{p}{180^\circ}\right) = 1.47\left(\frac{l}{2a}\right) \quad \Rightarrow \quad \frac{2a}{l} = \frac{(1.47)(90)}{p} = 42.1$

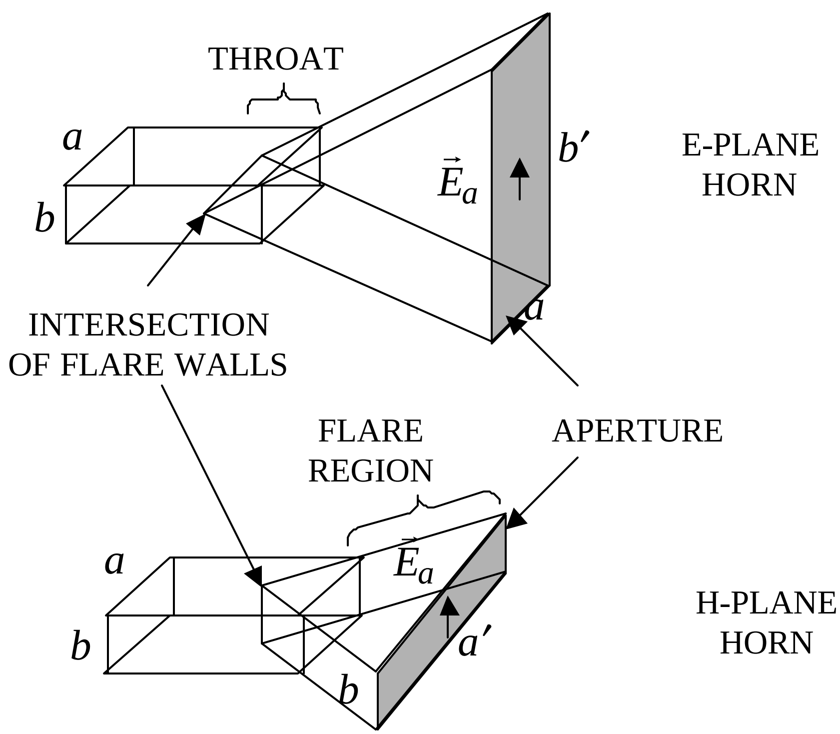
Elevation HPBW: $(25^\circ)\left(\frac{p}{180^\circ}\right) = 1.15\left(\frac{l}{2b}\right) \quad \Rightarrow \quad \frac{2b}{l} = \frac{(1.15)(7.2)}{p} = 2.64$

At 1 GHz the dimensions turn out to be 12.63m and 0.79m. The gain is

$$G = \frac{4p(42.1l)(2.64l)}{l^2}(0.833)(0.6522) = 758 = 28.8 \text{ dB}$$

Horn Antennas (1)

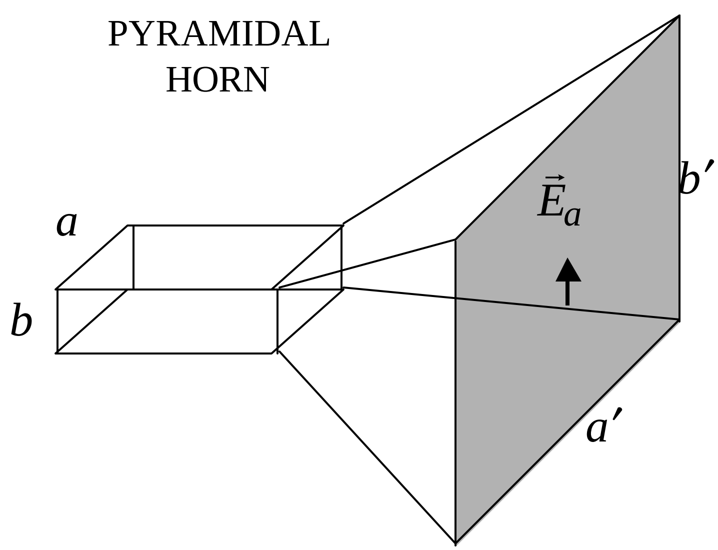
An earlier example dealt with an open-ended waveguide in an infinite ground plane. This configuration is not practical because the wave impedance in the guide is much different than the impedance of free space, and therefore a large reflection occurs at the opening. Very little energy is radiated; most is reflected back into the waveguide.

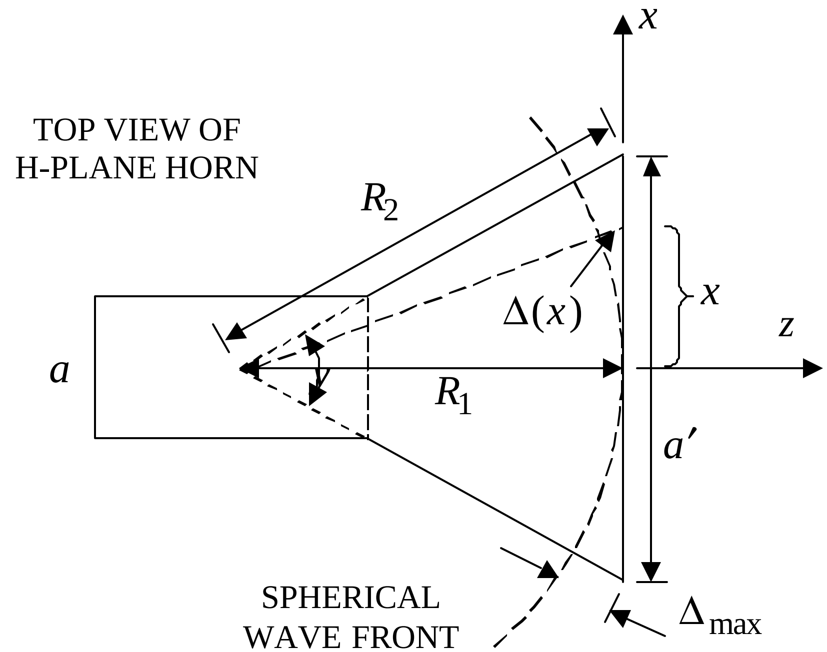


Flares are used to improve the match and increase the dimensions of the radiating aperture (to reduce beamwidth and increase gain). The result is a <u>horn antenna</u>. An E-plane horn has the top and bottom walls flared; an H-plane horn has the side walls flared. A pyramidal horn has all four walls flared, as shown on the next page.

Horn Antennas (2)

In most applications, the horn is not installed in a ground plane. Without a ground plane currents can flow on the outside surfaces of the horn, which modifies the radiation pattern slightly (mostly in the back hemisphere). We will neglect the exterior currents and compute the radiation pattern from the currents in the aperture only. The geometry of a H-plane horn is shown below.





Horn Antennas (3)

Assume that the waveguide has a TE_{10} mode at the opening. If the flare is long and gradual the following approximations will hold:

1. The amplitude of the field in the aperture is very close to a TE_{10} mode distribution.
2. The wave fronts at the aperture are spherical, with the phase center (spherical wave origin) at the intersection of the flare walls.

The deviation of the phase from that of a plane wave is given by $k\Delta(x)$, where

$$\Delta(x) = \sqrt{R_1^2 + x^2} - R_1 = R_1\left[\underbrace{\sqrt{1+\left(\frac{x}{R_1}\right)^2}}_{\approx 1+\frac{1}{2}\left(\frac{x}{R_1}\right)^2} - 1\right] \approx \frac{x^2}{2R_1}$$

The phase error depends on the square of the distance from the center of the aperture, and therefore is called a <u>quadratic phase error</u>. The electric field distribution in the aperture is approximately

$$\vec{E}_a = \hat{y}E_o \cos\left(\frac{px}{a'}\right)e^{-jkR(x)} = \hat{y}E_o \cos\left(\frac{px}{a'}\right)e^{-jk(R_1+\Delta(x))} \quad \rightarrow \quad \hat{y}E_o \cos\left(\frac{px}{a'}\right)e^{-jk\Delta(x)}$$

Horn Antennas (4)

The R_1 in the exponent has been dropped because it is a common phase that does not affect the far-field pattern. The equivalent magnetic current in the aperture is

$$\vec{J}_{ms} = -2\hat{z}\times\vec{E}_a = \hat{x}2E_o \cos\left(\frac{px}{a'}\right)e^{-jk\Delta(x)}$$

If the wave at the aperture is spherical (i.e., TEM) then the magnetic field is easily obtained from the electric field, and the equivalent electric current can be found

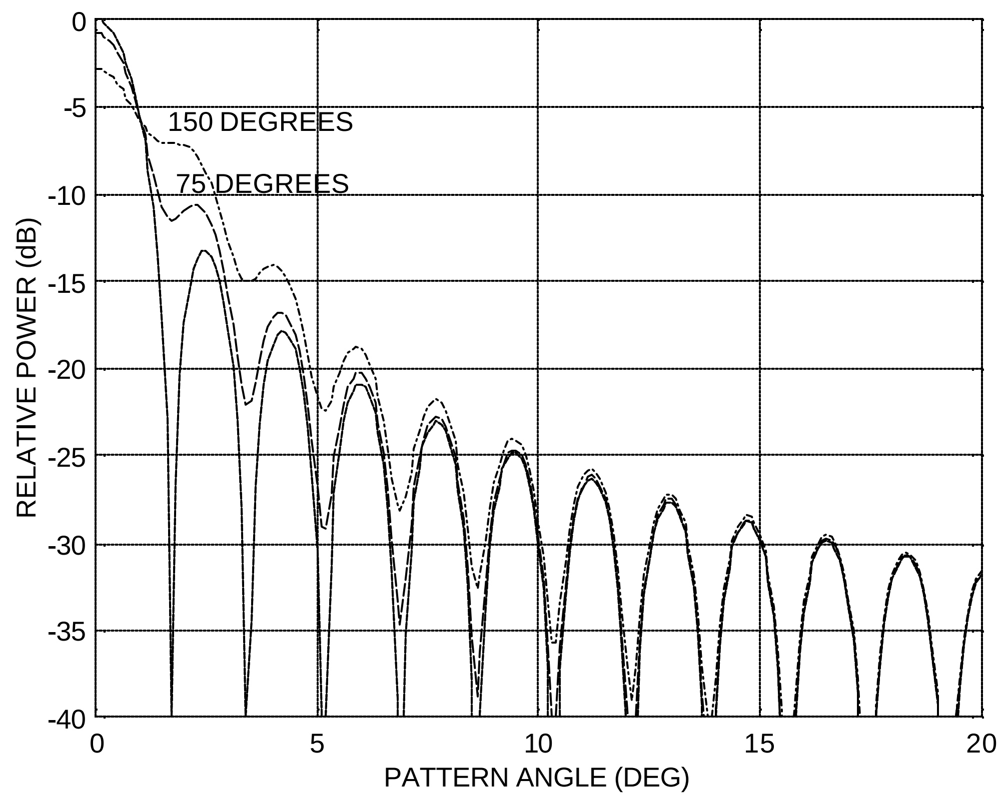
$$\vec{J}_s = 2\hat{z}\times\vec{H}_a = 2\hat{z}\times\frac{\hat{z}\times\vec{E}_a}{h} = -\hat{y}\,\frac{2E_o}{h}\cos\left(\frac{px}{a'}\right)e^{-jk\Delta(x)}$$

These currents are used in the radiation integral. Because of the presence of $k\Delta(x)$ in the exponential, the integrals cannot be reduced to a closed form result.

<u>The major tradeoff in the design of a horn</u>: in order to increase the directivity the aperture dimensions must be increased, but increasing the aperture dimensions also increases the quadratic phase error, which in turn decreases the directivity. What is the optimum aperture size?

Horn Antennas (5)

Patterns of a 10λ aperture with and without quadratic phase error. The phase error decreases the directivity and increases the beamwidth and sidelobe level.



Horn Antennas (6)

A maximum phase error of 45 degrees is one criterion used to limit the length of the flare:

$$k\Delta_{\max} \le p/4$$

$$\frac{2p}{l}\left(R_2 - R_1\right) \le \frac{p}{4}$$

$$\frac{2p}{l} R_2\left(1-\cos(y/2)\right) \le \frac{p}{4}$$

$$\frac{2p}{l}\,\frac{a'}{2\sin(y/2)}\left[1-\cos(y/2)\right] \le \frac{p}{4}$$

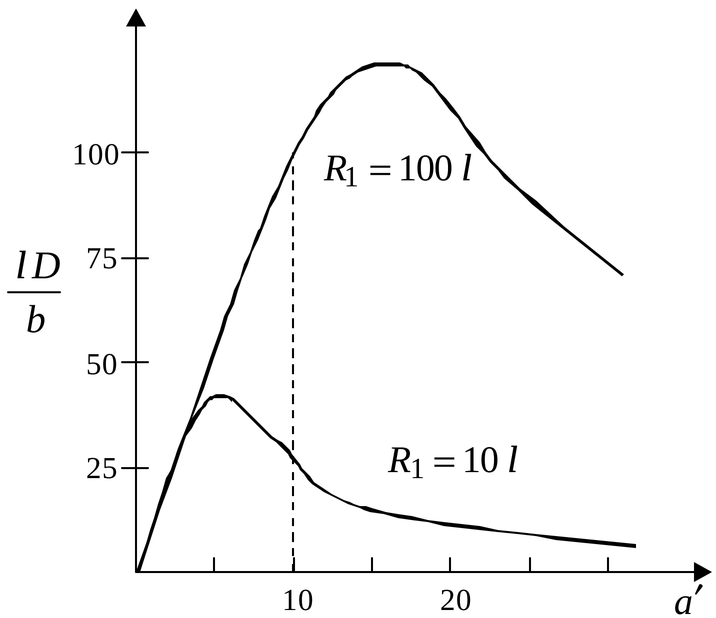
Use the identities $1-\cos(y/2) = 2\sin^2(y/4)$ and $\sin(y/2) = 2\sin(y/4)\cos(y/4)$

$$\frac{2\sin^2(y/4)}{2\sin(y/4)\cos(y/4)} \le \frac{l}{4a'} \quad \rightarrow \quad \tan(y/4) \le \frac{l}{4a'}$$

This is a good guideline for limiting the length of the flare based on pattern degradation, but does not necessarily give the optimum directivity.

Horn Antennas (7)

The optimum aperture width depends on the length of the flare, as shown below for an H-plane horn. A similar plot can be generated for an E-plane horn. (The separate factor on directivity is the reduction due to phase error.)



H-plane optimum:

$$a' = \sqrt{3 l R_{1_H}}$$

$$D_{\rm opt} = 10.2 \frac{a' b}{l^2} \left(\frac{1}{1.3} \right)$$

$$k\Delta_{\max} \approx 0.75 p$$

E-plane optimum:

$$b' = \sqrt{2\, l R_{1_E}}$$

$$D_{\rm opt} = 10.2 \frac{a b'}{l^2} \left(\frac{1}{1.25} \right)$$

$$k\Delta_{\max} \approx 0.5 p$$

Pyramidal optimum: $a' = \sqrt{3 l R_1},\ b' = \sqrt{2\, l R_1},\ D_{\rm opt} = 6.4 \frac{a' b'}{l^2} \quad (R_{1_H} = R_{1_E} = R_1)$

Horn Example

Example: An E-plane horn has $R_1 = 20\,l$ and $a = 0.5\,l$.

(a) The optimum aperture dimension for maximum directivity

$$b' = \sqrt{2\,l\,R_{1_E}} = l\sqrt{40} = 6.3\,l$$

(b) The flare angle for the optimum directivity

$$\tan(y\ /2) = \frac{b'/2}{R_1} = \frac{6.3\,l\ /2}{20\,l} = 0.1575$$

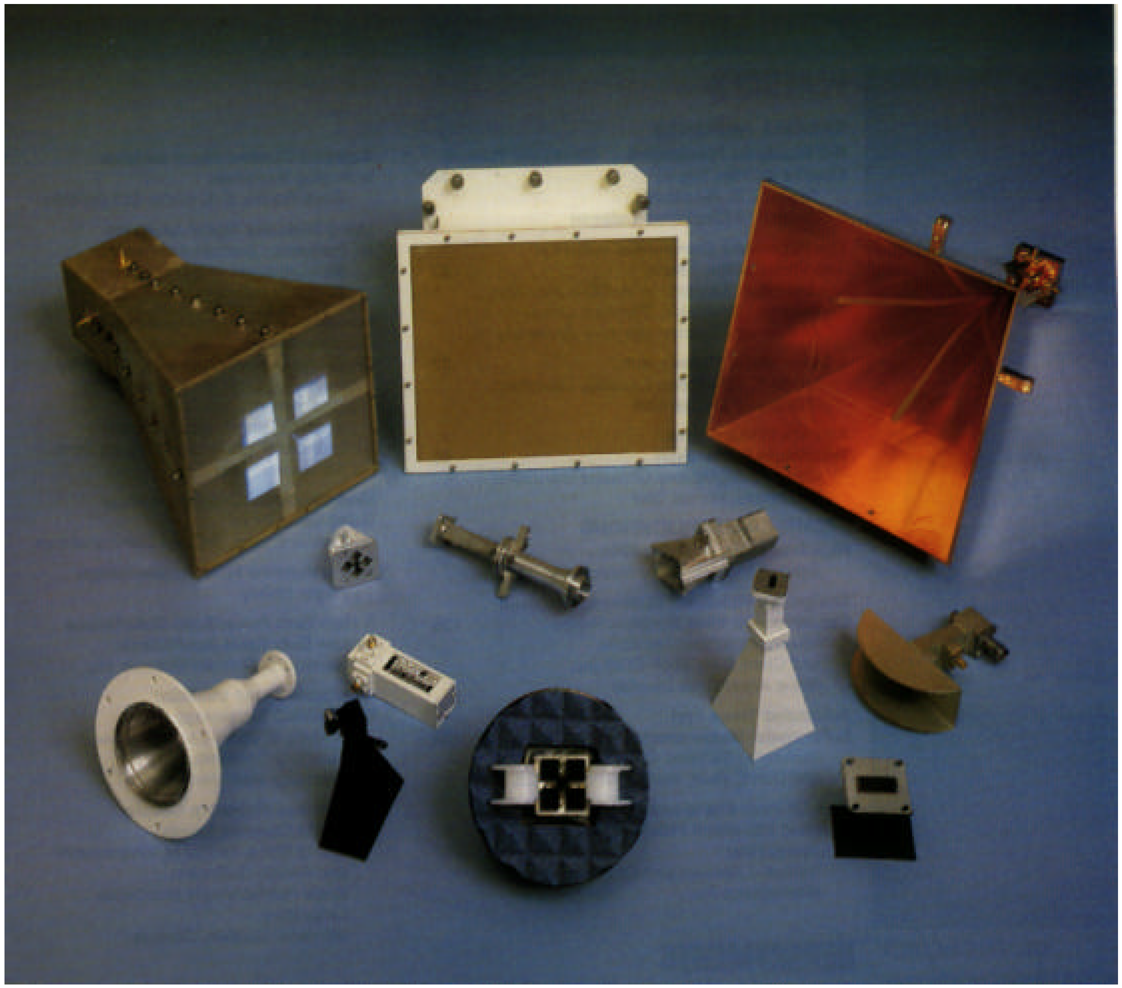
$$y\ /2 = 8.95^\circ$$

$$y\ = 17.9^\circ$$

(c) The optimum directivity is

$$D_{\text{opt}} = 10.2\frac{(0.5\,l\,)(6.3\,l\,)}{l^2}\left(\frac{1}{1.25}\right) = 25.7 = 14.1\ \text{dB}$$

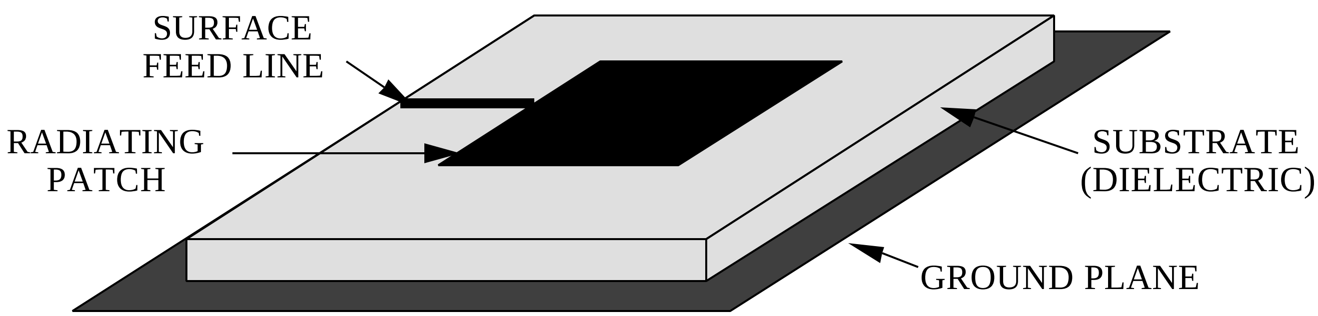
Several Types of Horn Antennas



Microstrip Patch Antennas (1)

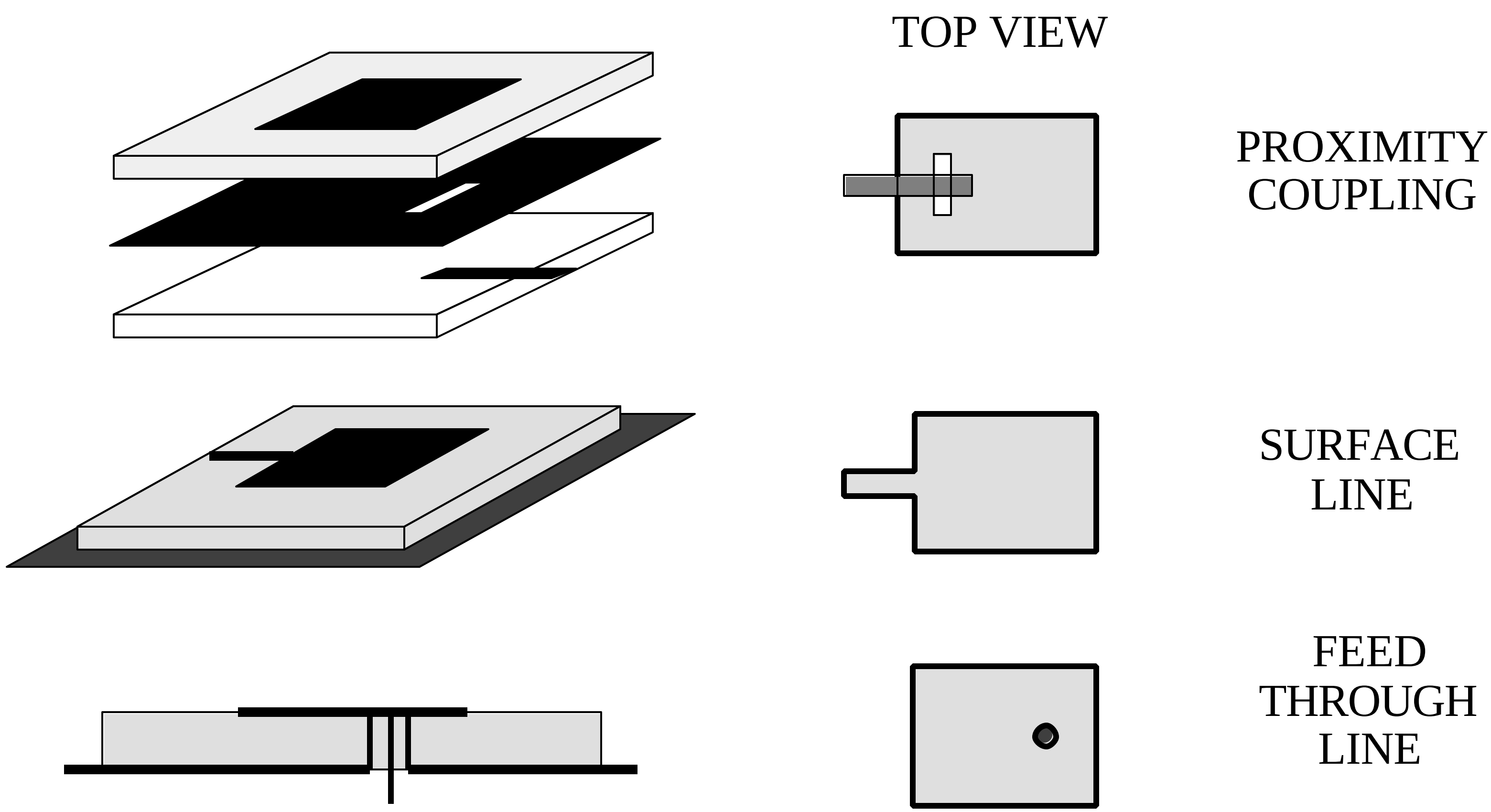
Microstrip patch antennas (or simply patch antennas) consist of a thin substrate of grounded dielectric this is plated on top with a smaller area of metal that serves as the element. The advantages and disadvantages include:

- Lend themselves to printed circuit fabrication techniques
- Low profile - ideal for conformal antennas
- Circular or linear polarization determined by feed configuration
- Difficult to increase bandwidth beyond several percent
- Substrates support surface waves
- Lossy



Microstrip Patch Antennas (2)

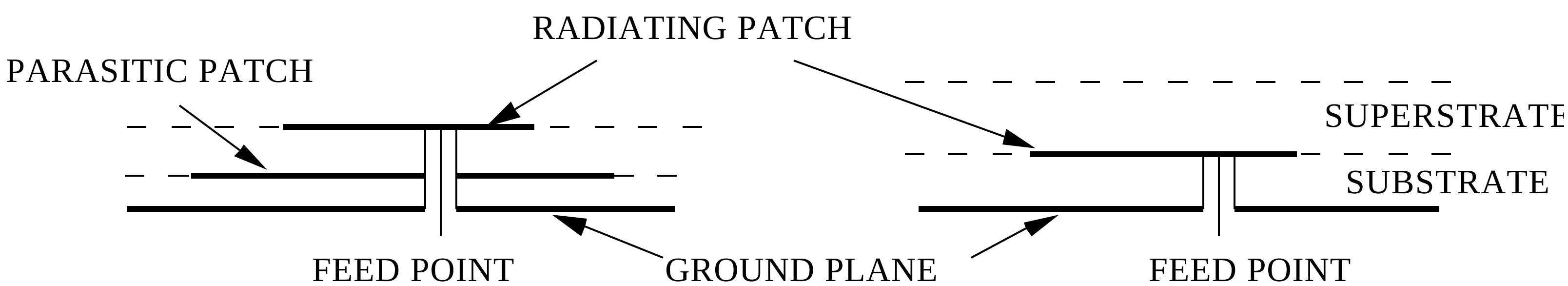
Several methods of feeding patch antennas are illustrated below:



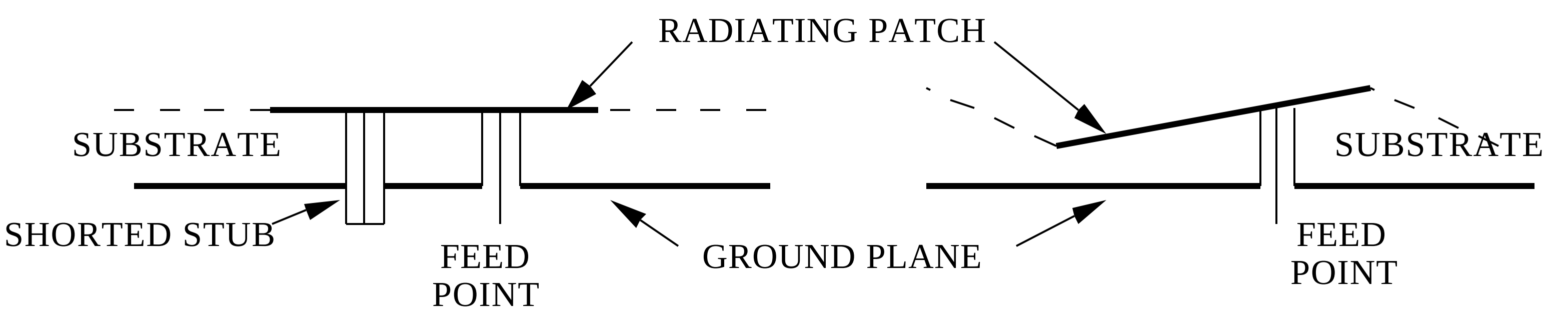
Microstrip Patch Antennas (3)

Several methods of broadbanding patch antennas are illustrated:

PARASITIC ELEMENTS SUPERSTRATES

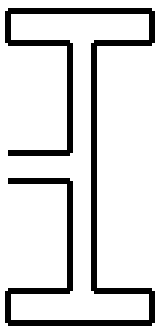


REACTIVE LOADING VARIABLE SUBSTRATES

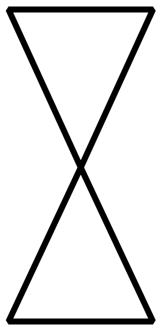


Microstrip Patch Antennas (4)

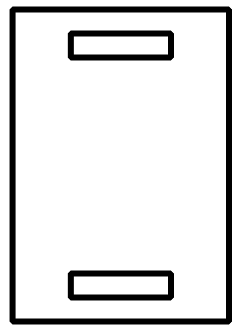
FOLDED
DIPOLE



BOW
TIE

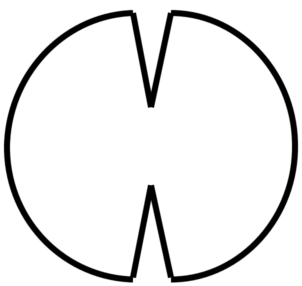


RECTANGULAR
SLOTTED

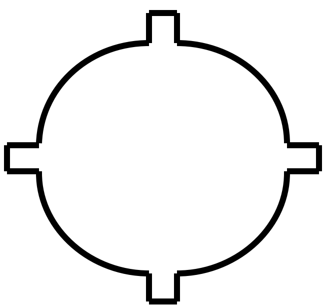


Modification of the basic element geometry can also provide some increased bandwidth.

CIRCULAR
SLOTTED



CIRCULAR
WITH "EARS"

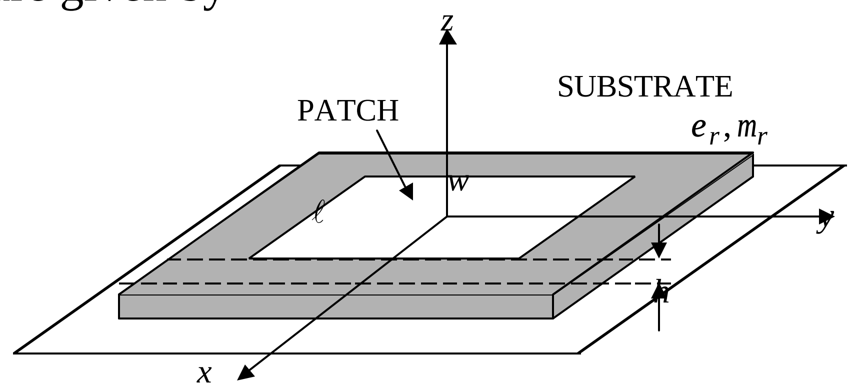


Microstrip Patch Antennas (5)

The rigorous formulas for a rectangular patch can be obtained from the Sommerfeld solution for an *x*-directed unit strength infinitesimal dipole located at the top of the substrate. The exact radiation patterns are given by

$$E_f = \left(\frac{jwm_o}{4p\,r} \right) \sin f \; e^{-jkr} F(q)$$

$$E_q = -\left(\frac{jwm_o}{4p\,r} \right) \cos f \; e^{-jkr} G(q)$$



where

$$F(q) = \frac{2\tan(k_1 h)}{\tan(k_1 h) - j(n_1(q)\sec q)/m_r}$$

$$G(q) = \frac{2\tan(k_1 h)\cos q}{\tan(k_1 h) - j(e_r \cos q)/n_1(q)}$$

and $k_1 = kn_1(q)$, $n_1(q) = \sqrt{n_1^2 - \sin^2 q}$, $n_1 = \sqrt{e_r m_r}$

Microstrip Patch Antennas (6)

The power radiated into space (assuming $h << l$) is

$$P_{\text{rad}} = 80\left(\frac{khpm_r}{l}\right)^2\left(1-\frac{1}{n_1^2}+\frac{2}{5n_1^4}\right)$$

However, some power may be captured by a surface wave. If the substrate is thin ($h << l$):

$$P_{\text{surf}} \approx \frac{60(khpm_r)^3}{l^2}\left(1-\frac{1}{n_1^2}\right)^3$$

The radiation efficiency is $e_{\text{rad}} \approx \dfrac{P_{\text{rad}}}{P_{\text{rad}}+P_{\text{surf}}}$. Define a new constant p that is a function of the ratio of the patch's radiated power to a Hertzian dipole's radiated power

$$p = \left[1+\frac{a_2}{20}(kw)^2+\frac{3a_4}{560}(kw)^4+\frac{b_2}{10}(k\ell)^2\right]\left(1-\frac{1}{n_1^2}+\frac{2}{5n_1^4}\right)$$

where $a_2 = -0.16605$, $a_4 = 0.00761$, and $b_2 = -0.09142$.

Microstrip Patch Antennas (7)

The input resistance is

$$R_{\text{in}} \approx \frac{90 e_{\text{rad}}}{p} m_r e_r \left(\frac{\ell}{w} \right)^2 \sin^2 \left(\frac{p\, x_o}{\ell} \right)$$

(x_o, y_o) is the location of the feed point. The bandwidth (defined as VSWR < 2) is

$$\text{BW} \approx \frac{16 p}{3\sqrt{2} e_r e_{\text{rad}}} \left(\frac{w}{\ell} \right) \left(\frac{h}{l} \right)$$

Example: Nonmagnetic substrate with $e_r = 2.2$, $f = 3.0$ GHz, $w/\ell = 1.5$, and $h/l = 0.025$. The length is chosen for resonance, $\ell = 0.0311$ m.

From the formulas presented, if the feed location is $(x_o = 0.0057, y_o = 0)$, then the input resistance is 43 ohms, the bandwidth approximately 0.037 (3.7%), and the radiation efficiency 0.913.

Reflector Antennas

Reflector systems have been used in optical devices (telescopes, microscopes, etc.) for centuries. They are a simple means of generating a large radiating aperture, which results in a high gain and narrow beamwidth. The most common is the “satellite dish,” a single surface parabolic reflector.

The advantages are:

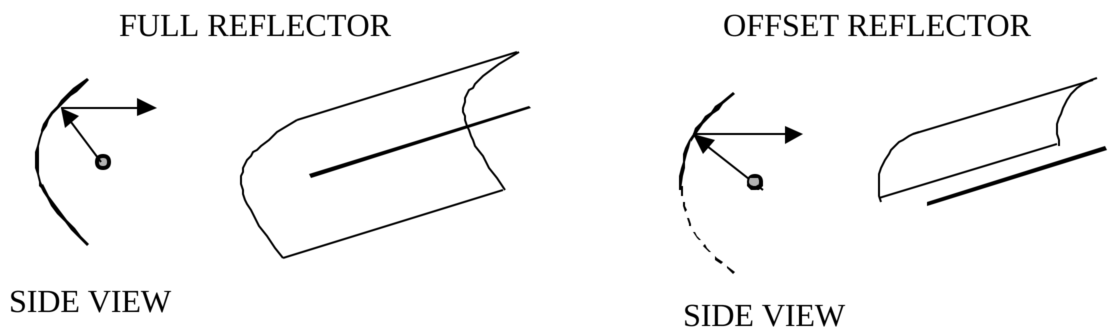
- Simple
- Broadband (provided that the feed antenna is broadband)
- Very large apertures possible

The disadvantages are:

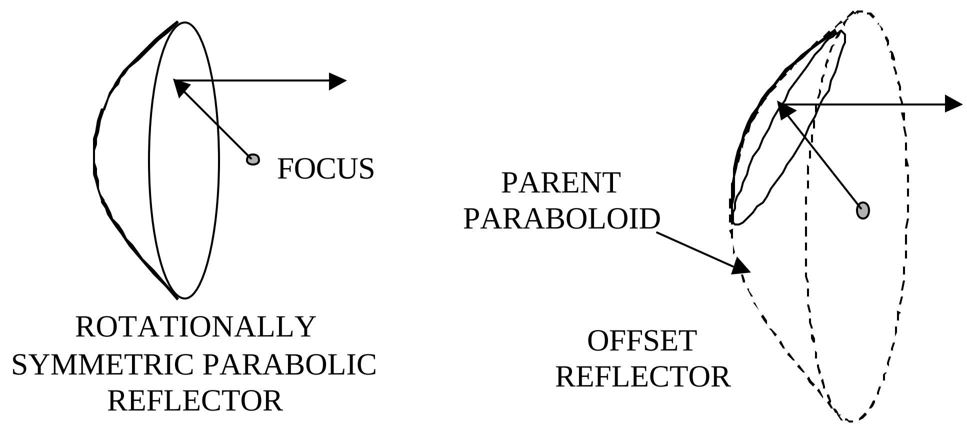
- Slow beam scanning
- Mechanical limitations (wind resistance, gravitational deformation, etc.)
- Surface roughness must be controlled
- Limited control of aperture illumination

Singly and Doubly Curved Reflectors

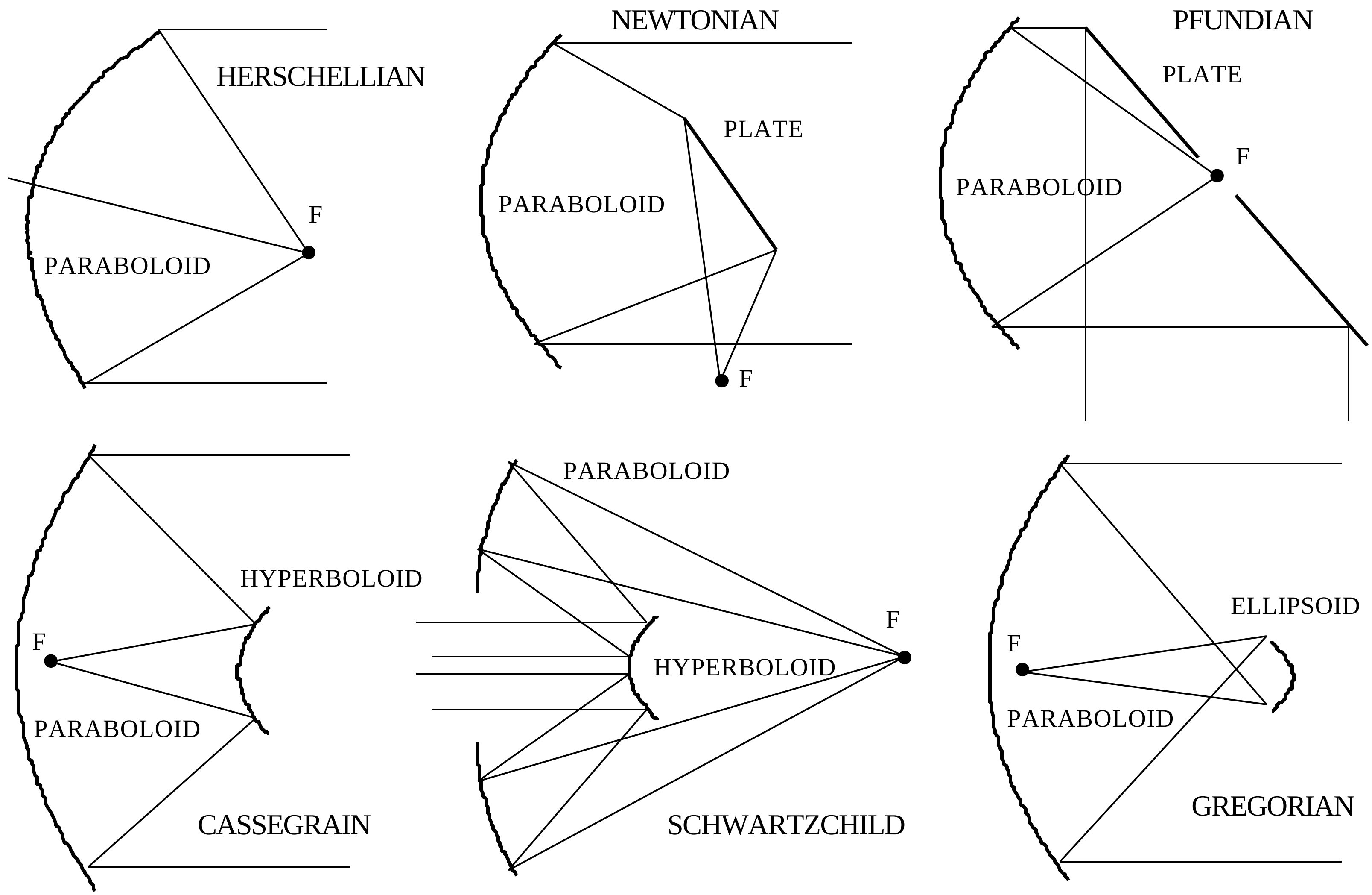
A singly curved reflector is generated by translating a plane curve (such as a parabola) along an axis. The radius of curvature in one dimension is finite; in the second dimension it is infinite. The focus is a line, and therefore a linear feed antenna is used.



A doubly curved reflector has two finite radii of curvature. The focus is a point. Spherical wave sources are used as feeds.



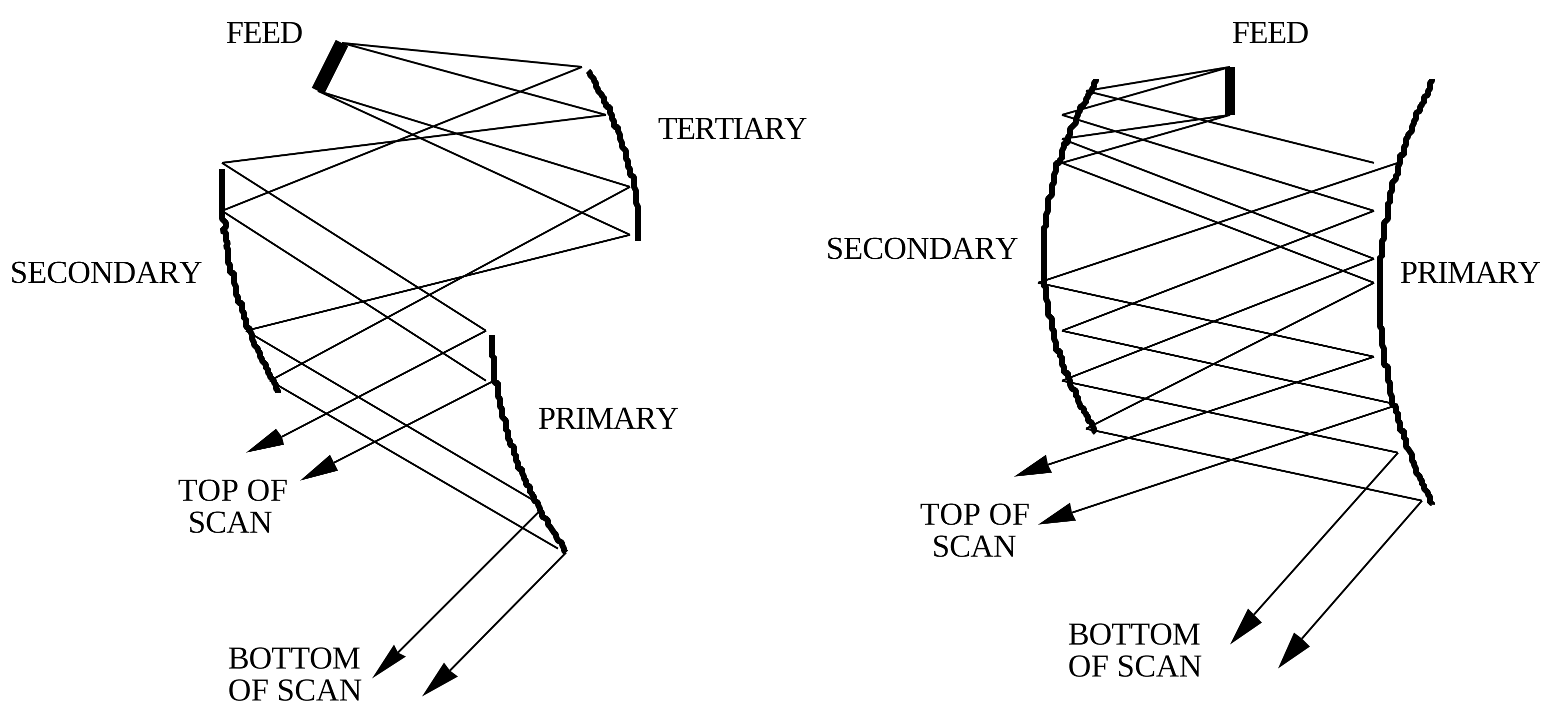
Classical Reflecting Systems



“Deep Space” Cassegrain Reflector Antenna

Multiple Reflector Antennas

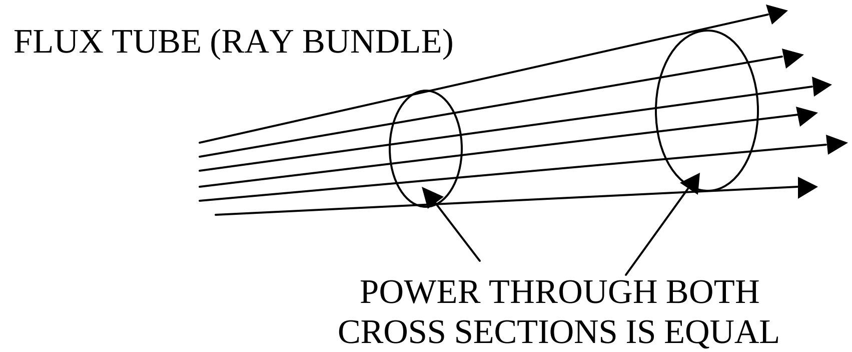
Dual reflecting systems like the Cassegrain and Gregorian are not uncommon. Some specialized systems have as many a four or five reflectors.



Geometrical Optics

Geometrical optics (GO) refers to the high-frequency ray tracing methods that have been used for centuries to design systems of lenses and reflectors. The postulates of GO are:

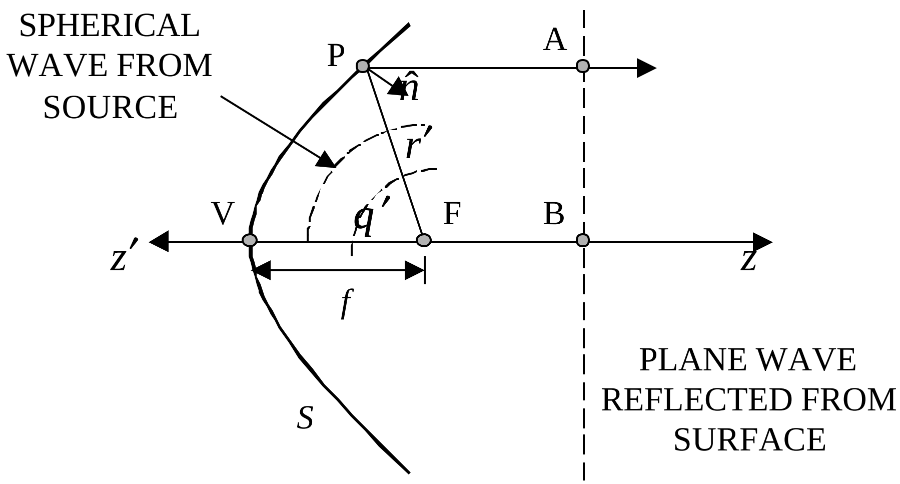
- Wavefronts are locally plane and TEM
- Wave directions are specified rays, which are vectors normal to the wavefronts (equiphase planes)
- Rays travel in straight lines in a homogeneous medium
- Polarization is constant along a ray in an isotropic medium
- Power contained in a bundle of rays (a flux tube) is conserved



- Reflection and refraction obeys Snell's law and is described by the Fresnel formulas
- The reflected field is linearly related to the incident field at the reflection point by a reflection coefficient (i.e., $E_{\mathrm{ref}} = E_{\mathrm{inc}}\Gamma$)

Parabolic Reflector Antenna (1)

What is the required shape of a surface so that it converts a spherical wave to a plane wave on reflection? All paths from O to the plane wave front AB must be equal:



F is the <u>focus</u>
V is the <u>vertex</u>
f is the <u>focal length</u>

$$\overline{\text{FP}} + \overline{\text{PA}} = \overline{\text{FV}} + \overline{\text{VB}}$$

$$\overline{\text{PA}} = \overline{\text{FP}}\cos q' + \overline{\text{FB}}$$

$$\overline{\text{VB}} = \overline{\text{FV}} + \overline{\text{FB}}$$

Plug in for $\overline{\text{VB}}$ and $\overline{\text{PA}}$

$$\overline{\text{FP}} + \left(\overline{\text{FP}}\cos q' + \overline{\text{FB}}\right) = \overline{\text{FV}} + \left(\overline{\text{FV}} + \overline{\text{FB}}\right)$$

$$\overline{\text{FP}}(1+\cos q') = 2\overline{\text{FV}}$$

$$r'(1+\cos q') = 2f$$

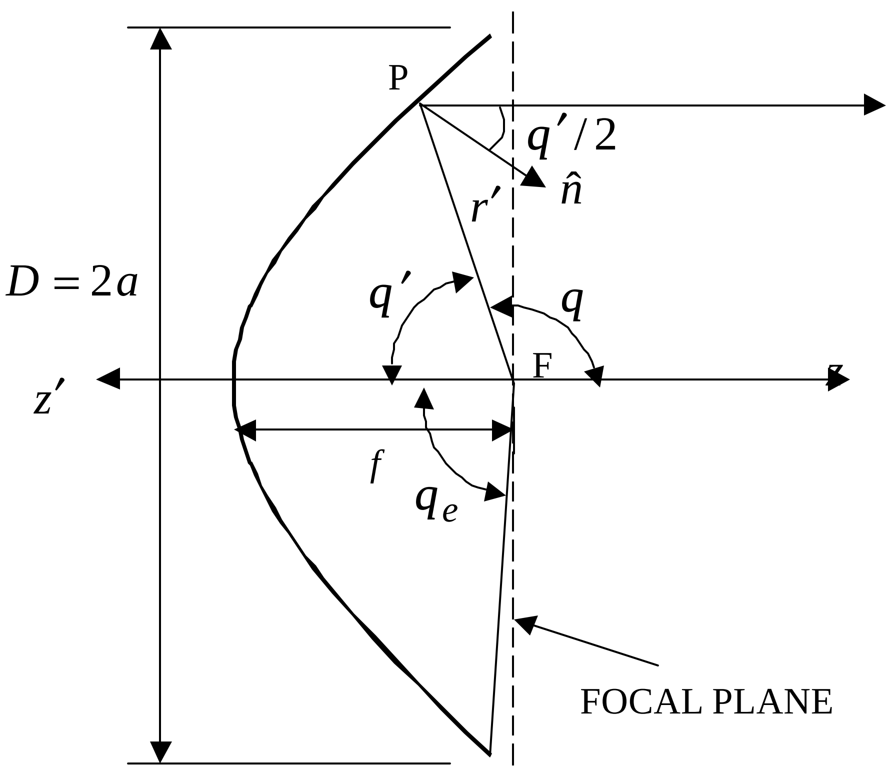
$$r' = 2f/(1+\cos q')$$

This is an equation for a parabola.

Parabolic Reflector Antenna (2)

The feed antenna is located at the focus. The design parameters of the parabolic reflector are the diameter *D*, and the ratio f / D. The edge angle is given by

$$q_e = 2\tan^{-1}\left[\frac{1}{4f / D}\right]$$

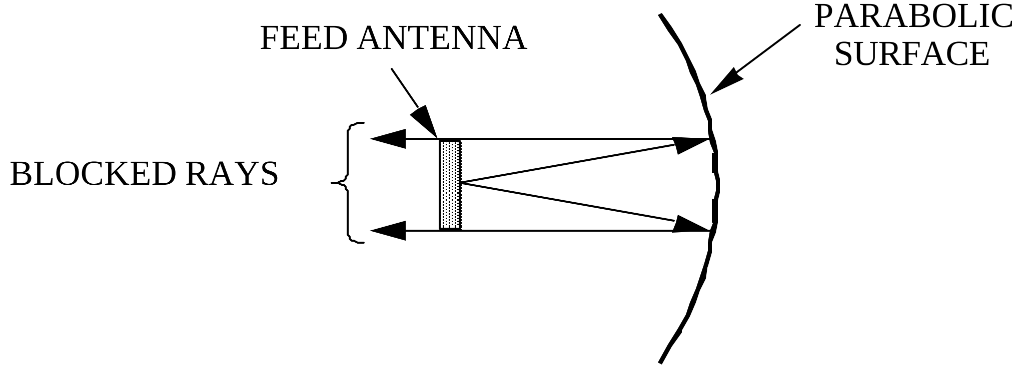


Ideally, the feed antenna should have the following characteristics:

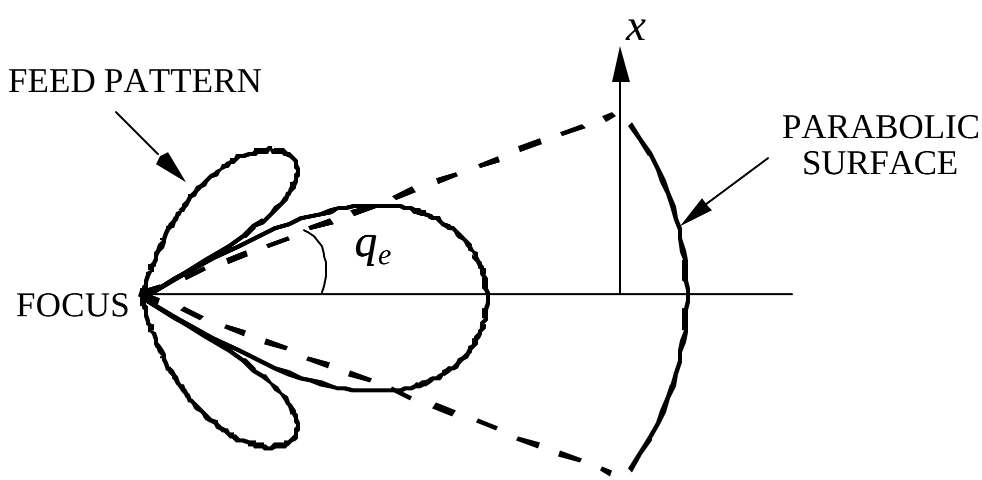
1. Maximize the feed energy intercepted by the reflector (small HPBW $\rightarrow$ large feed)
2. Provide nearly uniform illumination in the focal plane and no spillover (feed pattern abruptly goes to zero at q_e)
3. Radiate a spherical wave (reflector must be in the feed's far field $\rightarrow$ small feed)
4. Must not significantly block waves reflected off of the surface $\rightarrow$ small feed

Reflector Antenna Losses (1)

1. Feed blockage reduces gain and increases sidelobe levels (efficiency factor, e_b). Support struts can also contribute to blockage loss.



2. Spillover reduces gain and increases sidelobe levels (efficiency factor, e_s)



Reflector Antenna Losses (2)

3. Aperture tapering reduces gain (this is the same illumination efficiency that was encountered in arrays; efficiency factor, e_i)

4. Phase error in the aperture field (i.e., due to the roughness of the reflector surface, random phase errors occur in the aperture field, efficiency factor, e_p). Note that there are also random amplitude errors in the aperture field, but they will be accounted for in the illumination efficiency factor.

5. Cross polarization loss (efficiency factor, e_x). The curvature of the reflector surface gives rise to cross polarized currents, which in turn radiate a crossed polarized field. This factor accounts for the energy lost to crossed polarized radiation.

6. Feed efficiency (efficiency factor, e_f). This is the ratio of power radiated by the feed to the power into the feed.

This gain of the reflector can be written as

$$G = \frac{4pA}{l^2} e_a = \frac{4pA}{l^2} \underbrace{e_i e_p e_x}_{\equiv e_A} e_f e_s e_b$$

For reflectors, the product denoted as e_A is termed the aperture efficiency.

Example (1)

A circular parabolic reflector with $f/D = 0.5$ has a feed pattern $E(q') = \cos q'$ for $q' \le p/2$. The edge angle is

$$q_e = 2\tan^{-1}\left(\frac{1}{4f/D}\right) = (2)(26.56) = 53.1^\circ$$

The aperture illumination is

$$A(q') = \frac{e^{-jkr'}}{r'}\left|E(q')\right| = \frac{e^{-jkr'}}{r'}\left|\cos q'\right|$$

but $r' = 2f/(1+\cos q')$

$$\left|A(q')\right| = \frac{\cos q'(1+\cos q')}{2f}$$

The <u>edge taper</u> is the ratio of the field at the edge of the reflector to that at the center

$$\frac{\left|A(q_e)\right|}{\left|A(0)\right|} = \frac{\cos q_e(1+\cos q_e)/(2f)}{2/(2f)} = 0.4805 = -6.37 \text{ dB}$$

The feed pattern required for uniform amplitude distribution is

$$\frac{\left|A(q_e)\right|}{\left|A(0)\right|} = \frac{\left|E(q')\right|(1+\cos q')/(2f)}{\left|E(0)\right|/f} \equiv 1 \quad \rightarrow \quad \left|E(q')\right| = \frac{2}{(1+\cos q')} = \sec^2(q'/2)$$

Example (2)

The spillover loss is obtained from fraction of feed radiated power that falls outside of the reflector edge angles. The power intercepted by the reflector is

$$P_{\text{int}} = \int_0^{2\pi}\int_0^{\theta_e} \cos^2\theta' \sin\theta' d\phi\, d\theta' = 2\pi \int_0^{\theta_e} \cos^2\theta' \sin\theta' d\theta'$$

$$= -2\pi \left[\frac{\cos^3\theta'}{3}\right]_0^{\theta_e} = 0.522\pi$$

The total power radiated by the feed is

$$P_{\text{rad}} = \int_0^{2\pi}\int_0^{\pi/2} \cos^2\theta' \sin\theta' d\phi\, d\theta' = 2\pi \int_0^{\pi/2} \cos^2\theta' \sin\theta' d\theta'$$

$$= -2\pi \left[\frac{\cos^3\theta'}{3}\right]_0^{\pi/2} = 0.667\pi$$

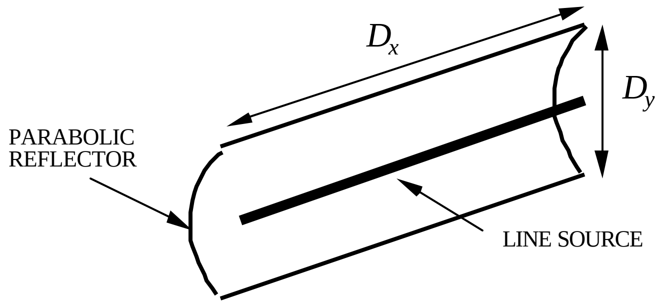
Thus the fraction of power collected by the reflector (spillover efficiency) is

$$e_s = P_{\text{int}} / P_{\text{rad}} = 0.522 / 0.667 = 0.78$$

The <u>spillover loss</u> in dB is $10\log(0.78) = -1.08$ dB.

Example (3)

A fan beam is generated by a cylindrical paraboloid fed by a line source that provides uniform illumination in azimuth and a $\cos(py'/D_y)$ distribution in elevation



The sidelobe levels (from Table 7.1of Slolnik or equivalent):

uniform distribution in azimuth (x),	SLL = 13.2 dB,	$e_{i_x} = 1$
cosine in elevation (y),	SLL = 23 dB,	$e_{i_y} = 0.81$

Find D_x and D_y for azimuth and elevation beamwidths of 2 and 12 degrees

$$q_{\text{el}} = 69l \; / \; D_y = 12^\circ \Rightarrow D_y = 5.75l$$

$$q_{\text{az}} = 51l \; / \; D_x = 2^\circ \Rightarrow D_x = 25.5l$$

The aperture efficiency is $e_i = (1)(.81)$ and the gain is

$$G = \frac{4pA_p}{l^2} e_i = \frac{4p\,(5.75l)(25.5l)}{l^2}(1)(0.81) = 1491.7 = 31.7$$

Calculation of Efficiencies (1)

Spillover loss can be computed from the feed antenna pattern. If the feed pattern can be expressed as $\vec{E}_f(r',q') = \sqrt{g(q')}\,\dfrac{e^{-jkr'}}{r'}\hat{e}_f$ where $g(q')$ gives the angular dependence and $\hat{e}_f$ denotes the electric field polarization, then the <u>spillover efficiency</u> is

$$e_s = \frac{\text{FEED POWER INTERCEPTED}}{\text{FEED POWER RADIATED}} = \frac{\displaystyle\int_0^{2p}\int_0^{q_e} \left|g(q')\right| \sin q'\, dq'\, df'}{\displaystyle\int_0^{2p}\int_0^{p} \left|g(q')\right| \sin q'\, dq'\, df'}$$

Example: What is the spillover loss when a dipole feeds a paraboloid with $f/D = 0.4$?

$$e_s = \frac{\displaystyle\int_0^{2p}\int_0^{64^\circ} \left|\sin^2 q'\right| \sin q'\, dq'\, df'}{\displaystyle\int_0^{2p}\int_0^{p} \left|\sin^2 q'\right| \sin q'\, dq'\, df'} = \frac{\left[\cos q' + \dfrac{\cos^3 q'}{3}\right]_0^{64^\circ}}{\left[\cos q' + \dfrac{\cos^3 q'}{3}\right]_0^{180^\circ}} = \frac{-1.3}{-2.667} = 0.488 = -3.1\text{ dB}$$

Calculation of Efficiencies (2)

The illumination efficiency (also known as tapering efficiency) depends on the feed pattern as well

$$e_i = 32\left(\frac{f}{D}\right)^2 \frac{\left|\int_0^{2p}\int_0^{q_e}\sqrt{g(q')}\,\tan(q'/2)\,dq'\,df'\right|^2}{\int_0^{2p}\int_0^{p}\left|g(q')\right|\sin q'\,dq'\,df'}$$

A general feed model is the function $g(q') = \begin{cases} 2(n+1)\cos^n q', & 0 \le q' \le p/2 \\ 0, & \text{else} \end{cases}$

The formulas presented yield the following efficiencies for this simple feed model:

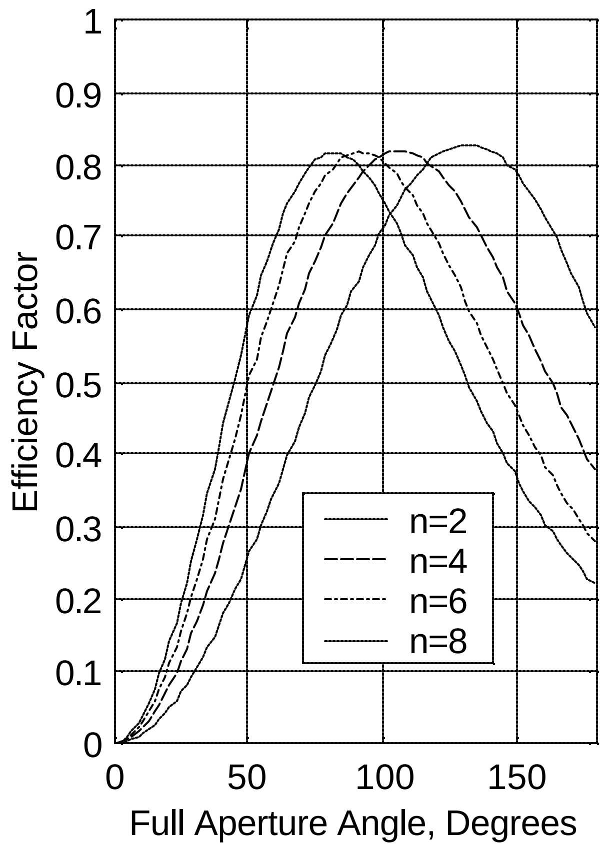
$$e_s = (n+1)\int_0^{q_e}\cos^n q' \sin q'\,dq' = 1 - \left[\cos\left(\frac{q_e}{2}\right)\right]^{n+1}$$

$$e_i = \left(\frac{f}{D}\right)^2 2(n+1)\left[\int_0^{q_e/2}\cos^{n/2} q' \tan(q'/2)\,dq'\right]^2$$

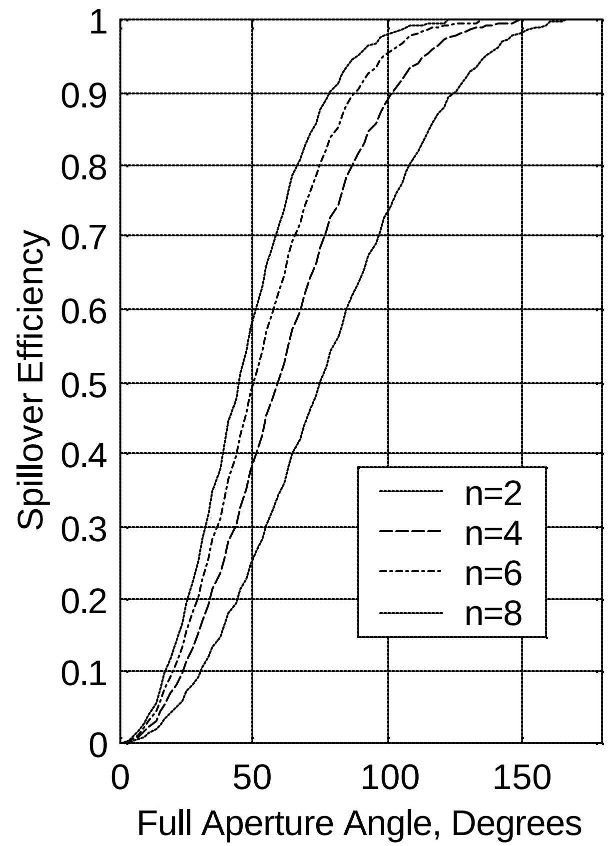
Cosine Feed Efficiency Factors

Efficiencies for a $\cos^n q'$ feed: (full aperture angle is $2q_e$)

Aperture efficiency ($e_i e_s$)



Spillover efficiency e_s



Feed Example

Given a reflector with $f/D = 0.5$ find n for a $\cos^n q'$ feed that gives optimum efficiency. Estimate the directivity of the feed antenna.

For $f/D = 0.5$ the edge angle is $q_e = 2\tan^{-1}\left(\frac{1}{4f/D}\right) = 53.1^\circ$. Therefore the full aperture angle is $2q_e = 106.3^\circ$. From the figure on the previous page, the curve with the maximum in the vicinity of 106.3° is n = 4, and therefore the feed exponent should be 4. The HPWB is

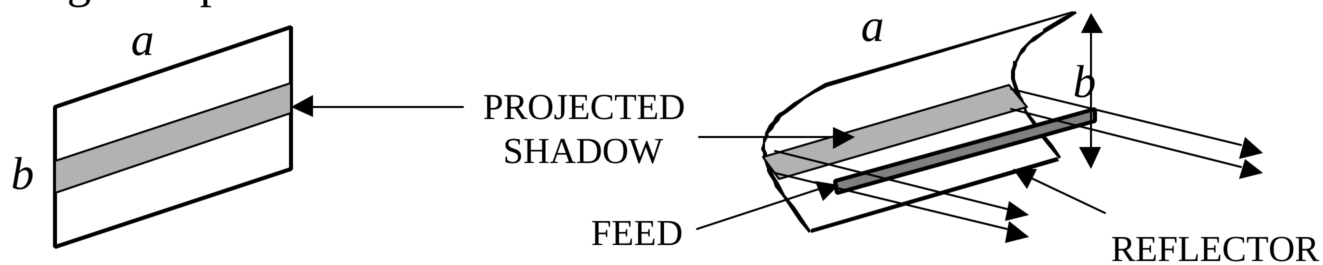
$$g(q') = \cos^4 q'_{HP} = 0.5$$
$$\cos q'_{HP} = 0.84$$
$$q'_{HP} = 32.8^\circ \quad \Rightarrow \quad \text{HPBW} = 65.5^\circ$$

We can use the formula for the directivity of the cosine pattern presented previously $D = 2(n+1) = 2(5) = 10 = 10$ dB. The approximate directivity formula can also be used

$$D \approx \frac{4p}{q_e f_a} = \frac{4p}{(1.14)^2} = 9.6 = 9.8 \text{ dB}$$

Calculation of Efficiencies (3)

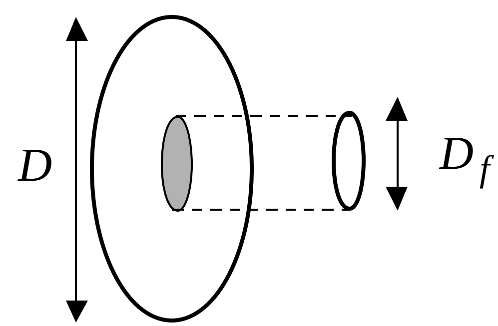
Feed blockage causes an additional loss in gain. For large reflectors, the null field hypothesis can be used to estimate the loss. Essentially it says that the current in the shadow of the feed projected on the aperture is zero. The shadow area is illustrated below for a rectangular aperture.



For a circular aperture, the area where nonzero currents exist is approximately

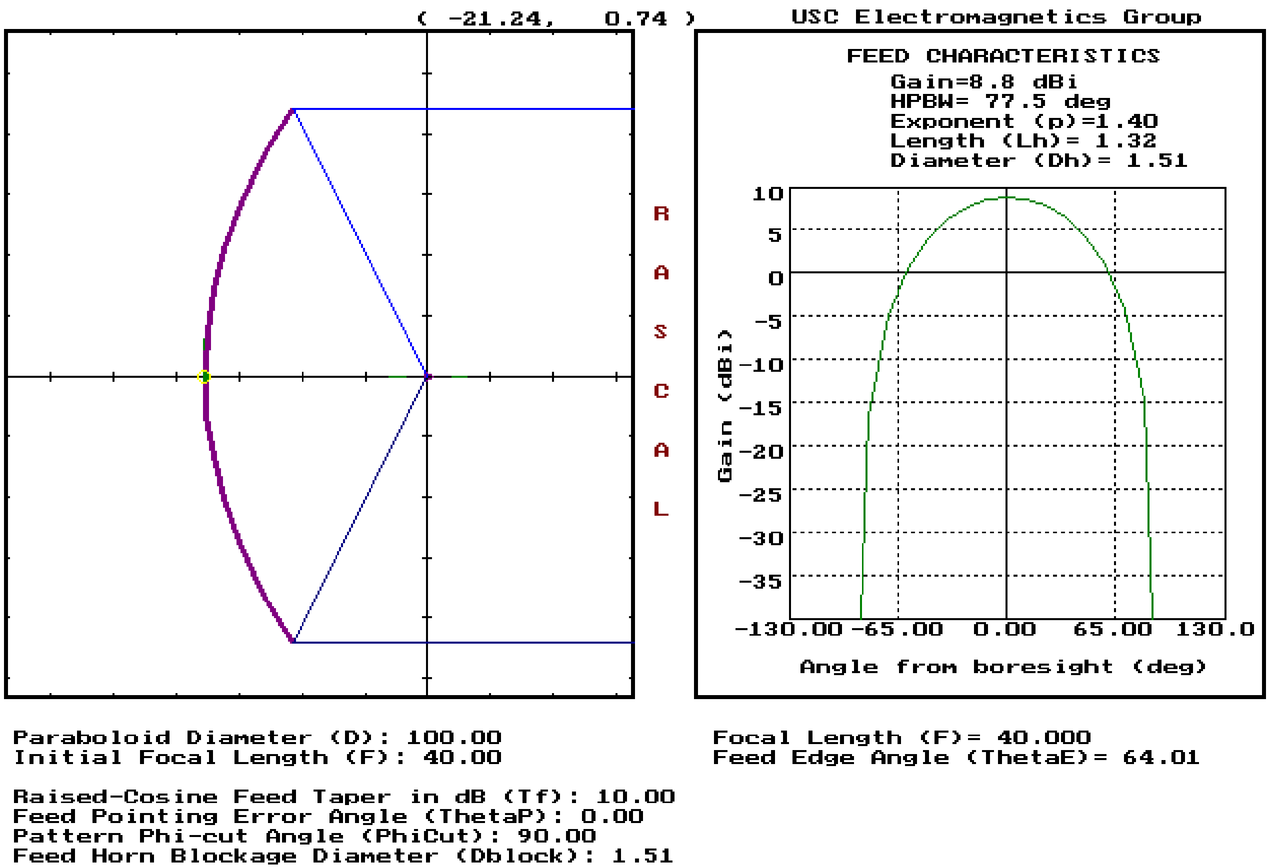
$$A_e \approx \pi\left(\frac{D}{2}\right)^2 - \pi\left(\frac{D_f}{2}\right)^2 = \frac{\pi}{4}\left(D^2 - D_f^2\right)$$

This assumes that all of the currents in the illuminated part of the aperture are uniform and in phase, which is not always the case. Both D and D_f should be much greater than the wavelength.



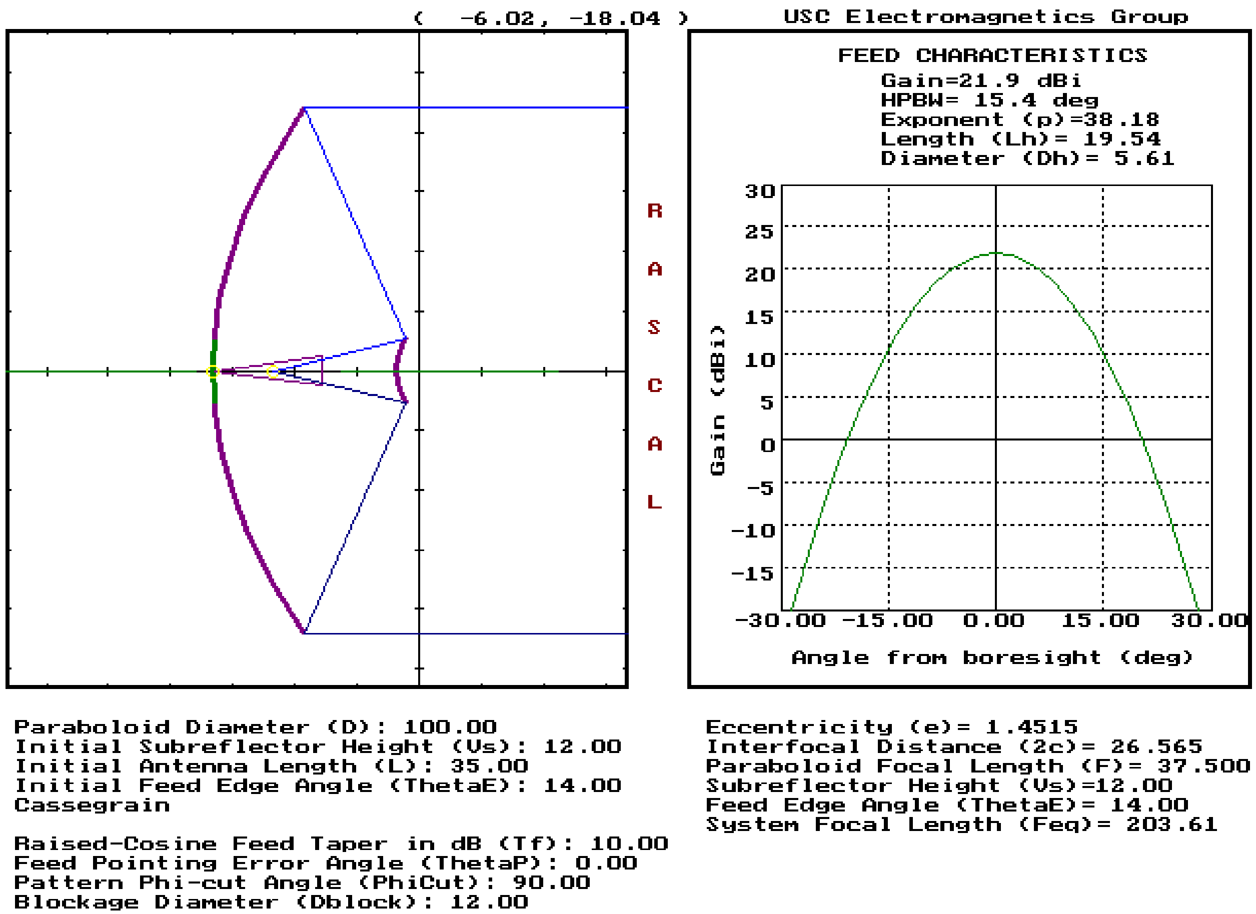
Reflector Design Using RASCAL (1)

Axially symmetric parabolic design:



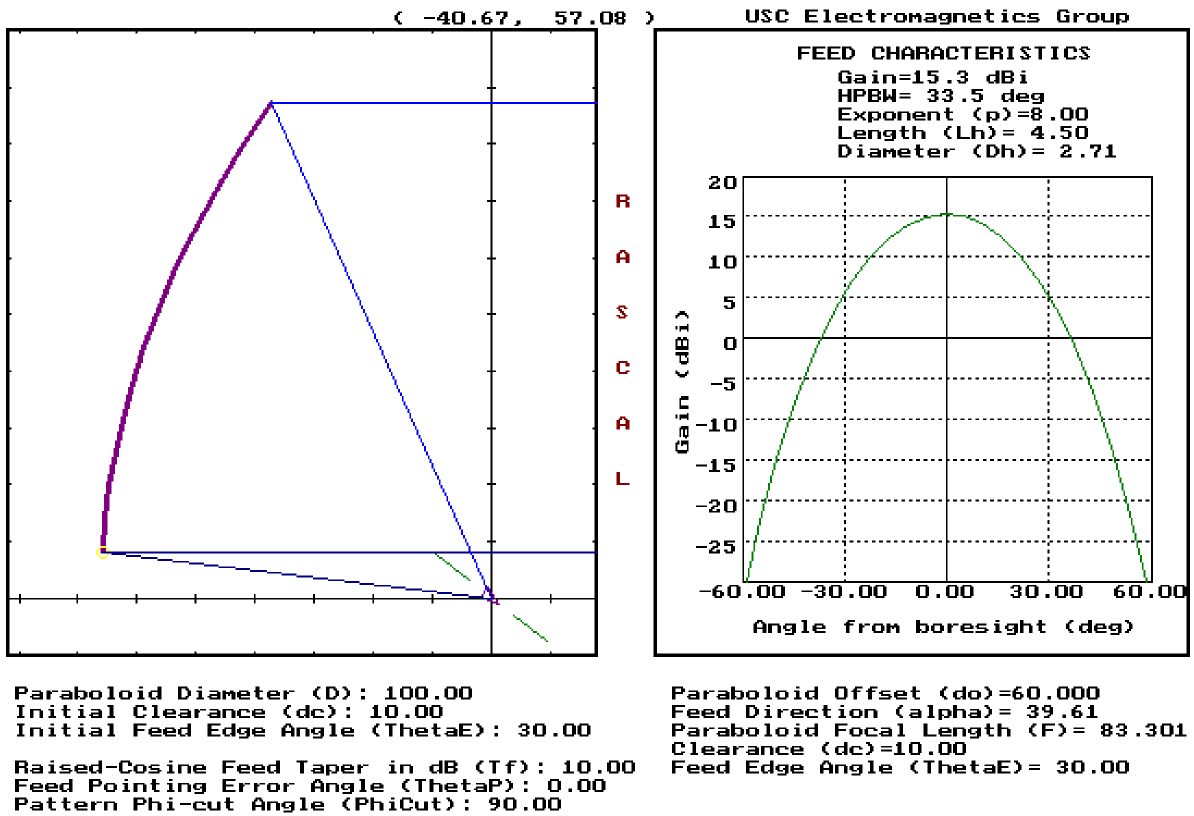
Reflector Design Using RASCAL (2)

Axially symmetric Cassegrain design:



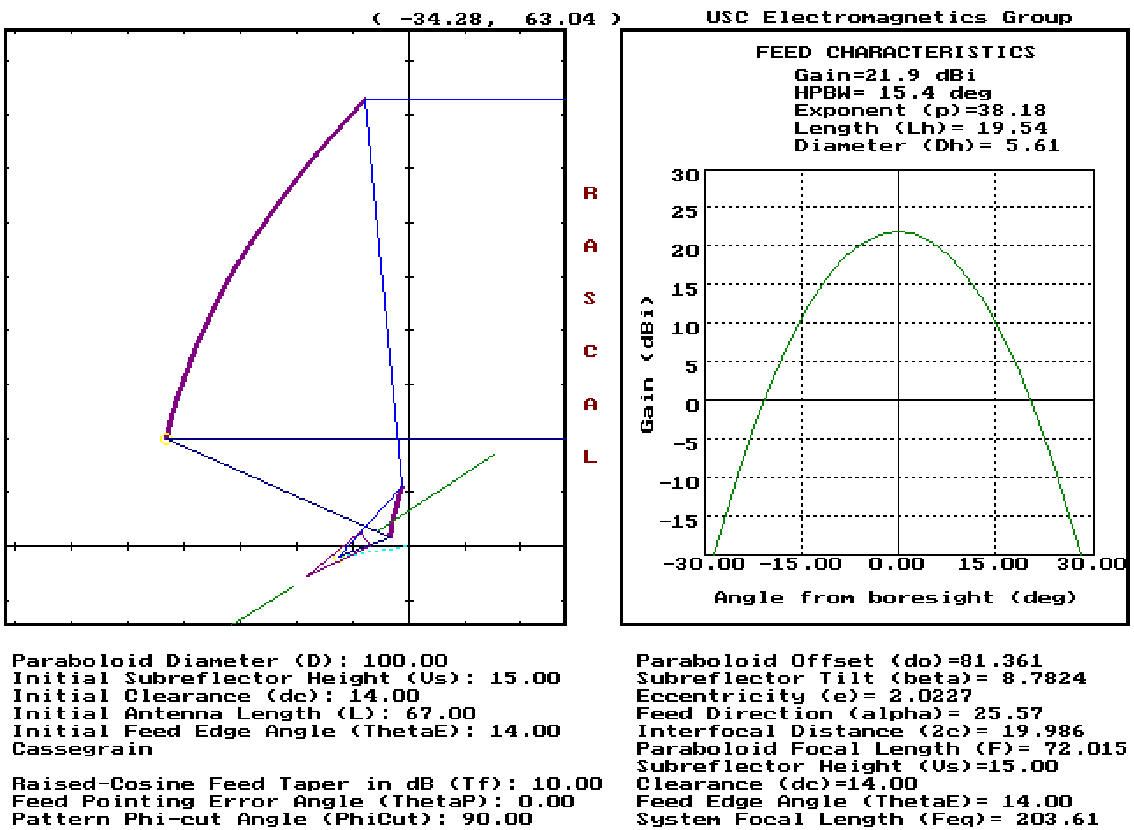
Reflector Design Using RASCAL (3)

Offset single surface paraboloid:



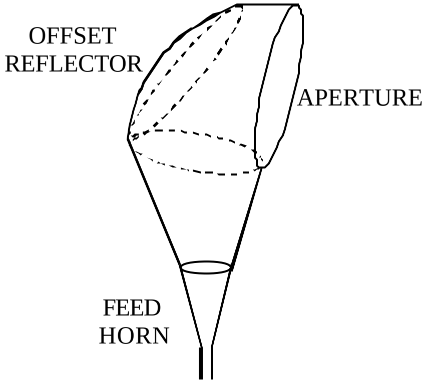
Reflector Design Using RASCAL (4)

Offset Cassegrain configuration:



Microwave Reflectors

Reflectors are used as microwave relays. The antennas with curved tops are horn-fed offset reflectors that are completely enclosed (called hog horns). The enclosures cut down on noise and interference cause by spillover. They also protect the antenna components from the elements (weather, birds, etc.) Axially symmetric reflector systems are also visible. They too are completely enclosed by a transparent radome.



Reflector Antenna Analysis Methods

Geometrical optics is used to design reflector surfaces, but is usually not accurate enough to use for predicting the secondary pattern (from the reflector). There are two common methods for computing the scattered field from the reflector:

1. Find or estimate the current induced on the reflector and use it in the radiation integrals.
   a) Rigorously calculate the current using a numerical approach such as the method of moments
   b) Estimate the current using the physical optics (PO) approximation

$$\vec{J}_s = 2\hat{n}\times\vec{H}_f(r',q',f')$$

   If the feed field is shadowed from a part of the surface, then the current is assumed to be zero on that part.

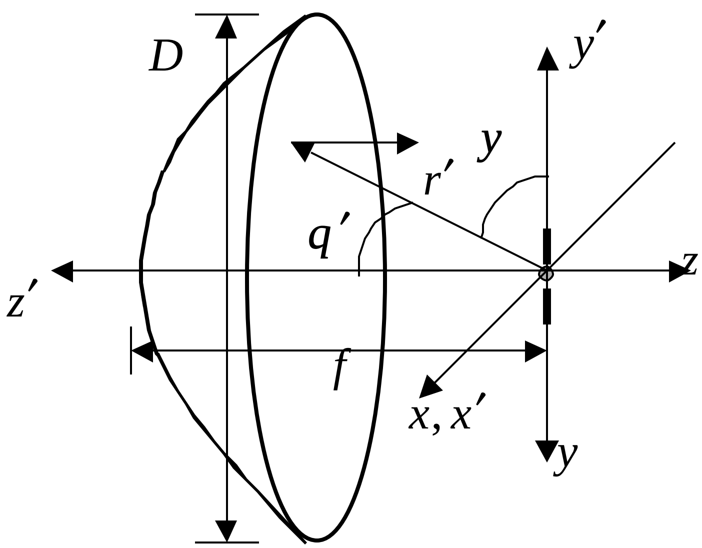
2. Equivalent aperture method. Find $\vec{E}_a$ in the aperture plane and compute an equivalent magnetic current that can be used in the radiation integral

$$\vec{J}_{ms} = -2\hat{n}\times\vec{E}_a(x',y',z')$$

Dipole Fed Parabola (1)

A dipole is used to feed a parabola. This is not practical because at least half of the the dipole's radiated power is directed in the rear hemisphere and misses the reflector (i.e., there is at least 3 dB of spillover loss). However, this example illustrates how crossed polarized currents and fields are generated. For the dipole aligned with the y axis

$$E_\phi(r', \theta', \phi') = \hat{e}_\phi \frac{jk\eta I\ell}{4\pi} \frac{e^{-jkr'}}{r'} \sin\psi \equiv E_o \hat{e}_\phi \frac{e^{-jkr'}}{r'} \sin\psi$$



There are two coordinate systems

$$x, y, z \rightarrow r, \theta, \phi \rightarrow \hat{r}, \hat{\theta}, \hat{\phi}$$

$$x', y', z' \rightarrow r', \theta', \phi' \rightarrow \hat{r}', \hat{\theta}', \hat{\phi}'$$

$\cos\psi$ is just the y' direction cosine

$$\cos\psi = \hat{r}' \bullet \hat{y}' = \sin\theta' \sin\phi'$$

Because ψ is the angle from the dipole axis, the dipole pattern depends on

$$\sin\psi = \sqrt{1 - \cos^2\psi} = \sqrt{1 - \sin^2\theta' \sin^2\phi'}$$

Dipole Fed Parabola (2)

If the reflector is in the far field of the dipole, then the electric field vector will have only $\hat{\theta}', \hat{\phi}'$ components

$$\hat{e}_f = \underbrace{\hat{r}' \sin\theta' \sin\phi'}_{\text{DROP THIS}} + \hat{\theta}' \cos\theta' \sin\phi' + \hat{\phi}' \cos\phi'$$

Re-normalizing the vector

$$\hat{e}_f = \frac{\hat{\theta}' \cos\theta' \sin\phi' + \hat{\phi}' \cos\phi'}{\sqrt{\cos^2\theta' \sin^2\phi' + \cos^2\phi'}} = \frac{\hat{\theta}' \cos\theta' \sin\phi' + \hat{\phi}' \cos\phi'}{\sqrt{1 - \sin^2\theta' \sin^2\phi'}} = \frac{\hat{\theta}' \cos\theta' \sin\phi' + \hat{\phi}' \cos\phi'}{\sin\psi}$$

which we rearrange to find

$$\hat{e}_f \sin\psi = \hat{\theta}' \cos\theta' \sin\phi' + \hat{\phi}' \cos\phi'$$

After the spherical wave is reflected from the parabola, a plane wave exists and there is no $1/r$ dependence. The field in the aperture is the field reflected from the parabola. At the reflector, the tangential components cancel and the normal components double

$$\left(\vec{E}_i + \vec{E}_r\right)_{\text{norm}} = 2\left(\vec{E}_i\right)_{\text{norm}} = 2(\hat{n} \bullet \vec{E}_i)\hat{n}$$

Dipole Fed Parabola (3)

Therefore,

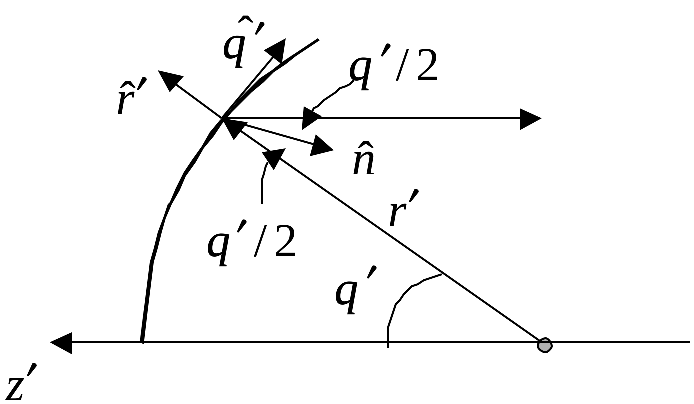
$$\vec{E}_a = \vec{E}_r = 2(\hat{n} \bullet \vec{E}_f)\hat{n} - \vec{E}_f$$

which is nothing more than a vector form of Snell's Law. The normal at a point on the reflector surface is given by

$$\hat{n} = -\hat{r}' \cos\frac{q'}{2} + \hat{q}' \sin\frac{q'}{2}$$

After some math, which involves the used of several trig identities, the aperture field is

$$\vec{E}_a(r', q', f') = E_o \frac{e^{-jkr'}}{r'} \left\{ \hat{x} \cos f' \sin f' (1 - \cos q') - \hat{y}(\cos q' \sin^2 f' + \cos^2 f') \right\}$$



The magnetic current in the aperture is

$$\vec{J}_{ms} = -2\hat{n} \times \vec{E}_a = -2\hat{z} \times \vec{E}_a$$

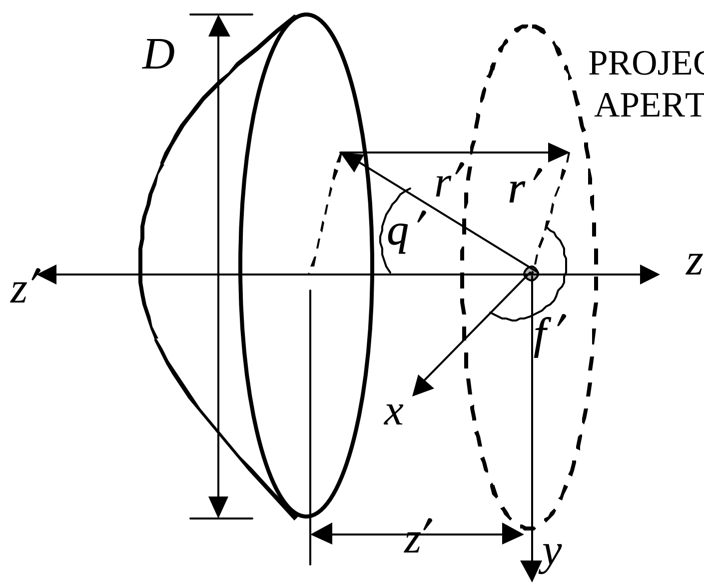
This current exists over a circular aperture, and it is used in the radiation integral to get the far field. Since the integration is over a circular region, it is convenient to use polar coordinates, (r', f')

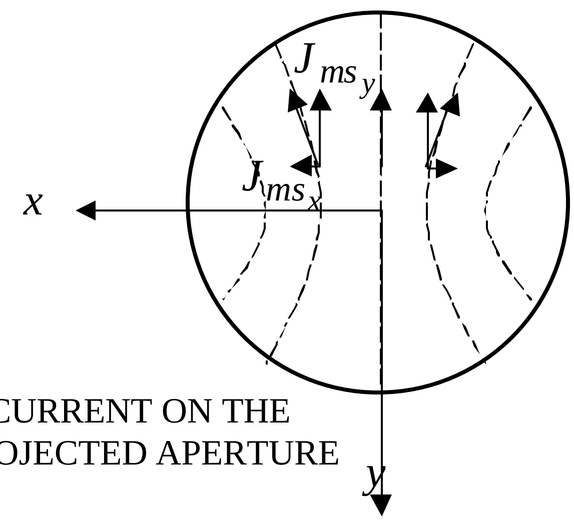
Dipole Fed Parabola (4)

The important characteristic of the aperture field is that there are both *x* and *y* components, even though the feed dipole is purely *y* polarized. Since the radiation integral has the form

$$E_{\left\{ q \atop f \right\}}(r,q,f) = \frac{jkh}{4p}\frac{e^{-jkr}}{r}\int_0^{2p}\int_0^{D/2}\left(\frac{\vec{J}_{ms}\times\hat{r}}{h}\right)\bullet\left\{ \hat{q} \atop \hat{f} \right\} e^{-jk\hat{r}\bullet\vec{r}'}\, r'\,dr'\,df'$$

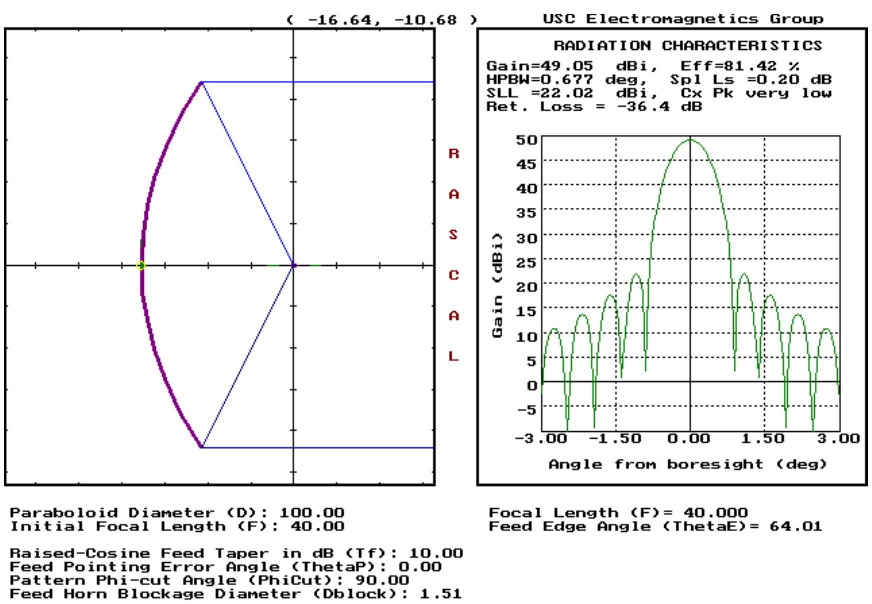
the *x* directed currents result in a crossed polarized far field component.

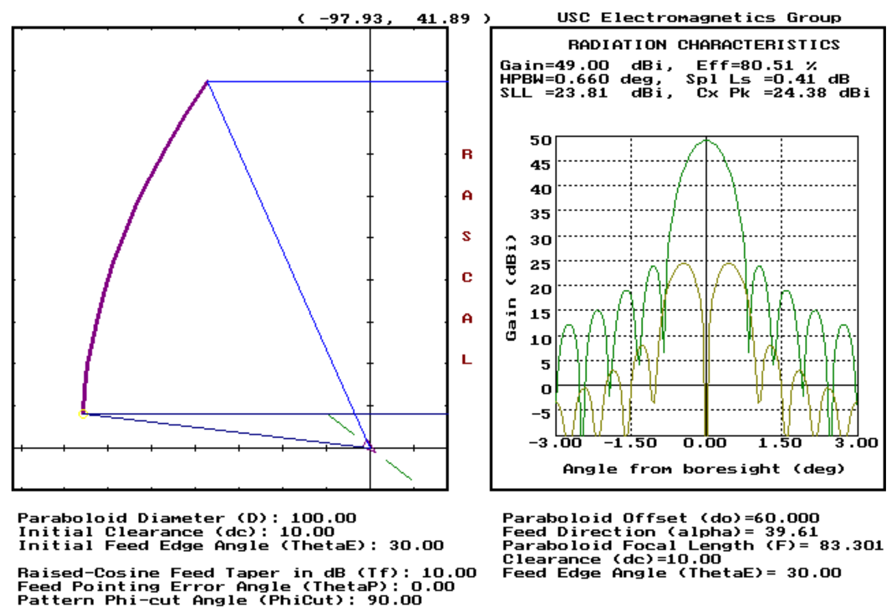




Crossed Polarized Radiation

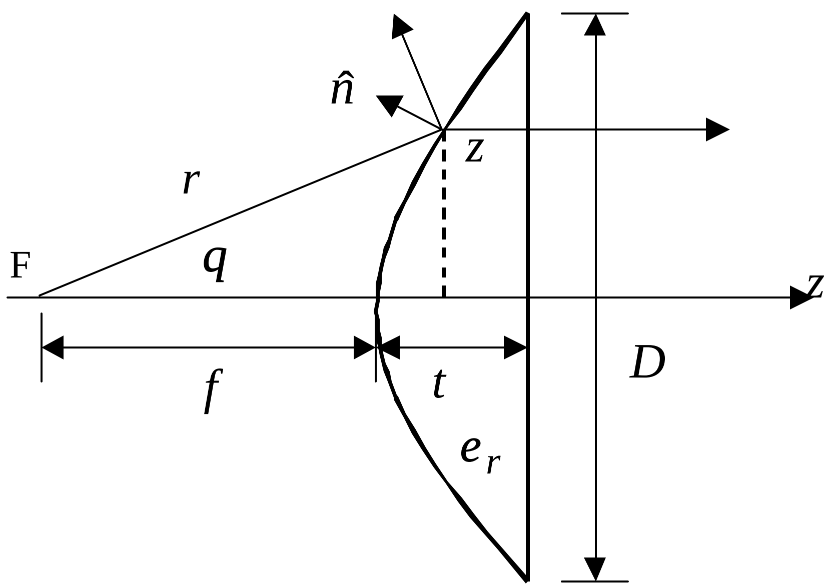
Below is a comparison of the crossed polarized radiation in the principal plane of a axially symmetric parabolic reflector to that of an offset parabolic reflector. In the symmetric case, the radiation from the crossed polarized components cancel when the observation point is in the principal plane. Note that the feed is not a dipole, but a raised cosine.





Lens Antennas (1)

Lens antennas are also based on geometrical optics principles. The major advantage of lenses over reflectors is the elimination of blockage. Lenses can be constructed the same way at microwave frequencies as they are at optical frequencies. A dielectric material is shaped to provide equal path lengths from the focus to the aperture, as illustrated below.



It is important to keep the reflection at the air/dielectric boundary as small as possible. The wavelength in the dielectric is $\lambda = \lambda_o / n$, where $n = \sqrt{\epsilon_r}$ is the index of refraction.

The axial path length is $f+nt$. The path length along the ray shown is $r+nz$. Since they must be equal,

$$f + t = r\cos\theta + z$$
$$t = r\cos\theta + z - f$$

Inserting t back in the original equation:

$$f + r\cos\theta + z - f = r\cos\theta + z$$

Lens Antennas (2)

Solving for r gives

$$r = \frac{f\,(n-1)}{n\cos\theta - 1}$$

which is the equation for a hyperbola.

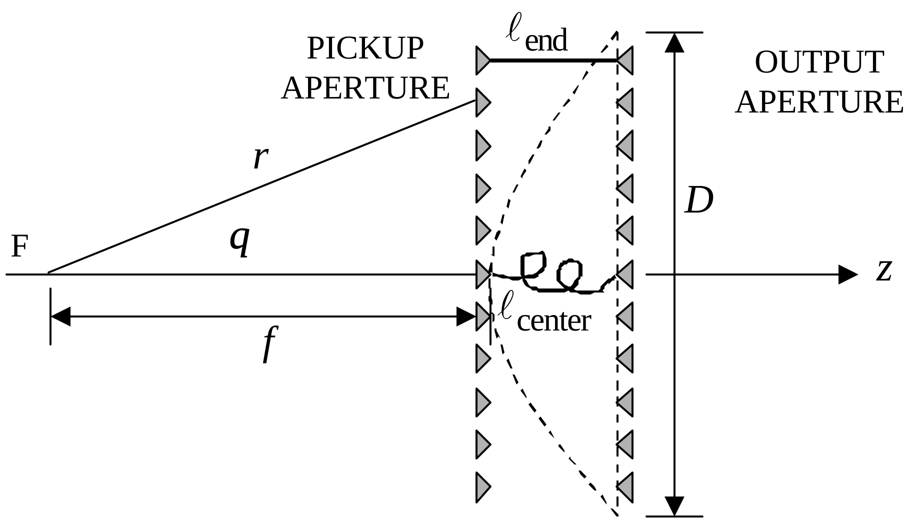
There are several practical problems with “optical type” lenses at microwave frequencies.

1. For a high gain a large D is required, yet the focal length must be small to keep the overall antenna volume small. Lenses can be extremely heavy and bulky.
2. Hyperoloids are difficult to manufacture, so a spherical approximation is often used for the lens shape. The sphere’s deviation from a hyperbola results in phase errors called <u>aberrations</u>. The errors distort the far field pattern similar to quadratic phase errors in horns.
3. Reflection loss occurs at the air/dielectric interface. There are also multiple reflections inside of the lens that cause aperture amplitude and phase errors.
4. As in the case of reflectors, there is spillover, non-uniform amplitude at the aperture, and crossed-polarized far fields.

Special design tricks can be employed at microwave frequencies. Since the wavelength is relatively large compared to optical case, a sampled version of the lens is practical.

Lens Antennas (3)

A sampled version of a lens would use two arrays placed back to back. The array of pickup elements receives the feed signal and transmits it to the second array at the output aperture. The cable between the elements provides the same phase shift that a path through a solid dielectric would provide. In fact, there is no need to curve the pickup array aperture. A plane surface can be used and any phase difference between the curved and plane surfaces are then included in the phase of the connecting cable.



In a dielectric lens, the shortest electrical path length ($k\ell_{end}$) is at the edge and the longest electrical path length ($k\ell_{center}$) is in the center. Therefore the cable in the center must be longer than the cable at the edge. Phase shifters could be inserted between the arrays to scan the output aperture beam. This approach is referred to as a <u>constrained lens</u> (i.e., the signal paths are constrained to cables).

Lens Antennas (4)

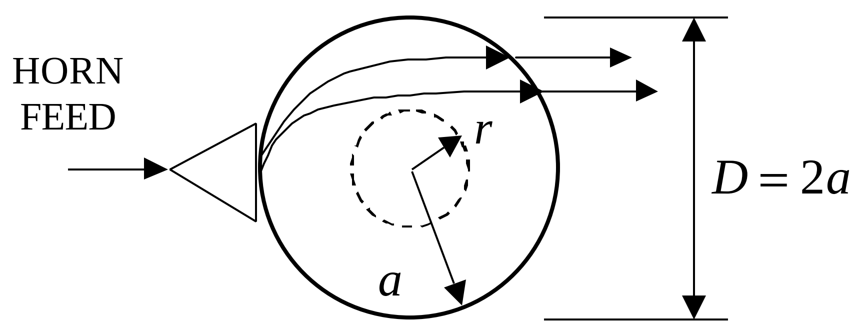
Constrained lenses still suffer spillover loss. However, the aperture surfaces can be planar (rather than hyperbolic or spherical). Generally they are much lighter weight than a solid lens.

Conventional reflectors and lenses must be scanned mechanically; that is, rotated or physically pointed. A limited amount of scanning can be achieved by moving the feed off of the focus. However, the farther the feed is displaced from the focal point, the larger the aperture phase deviation from a plane wave. This type of scanning is limited to just a few degrees.

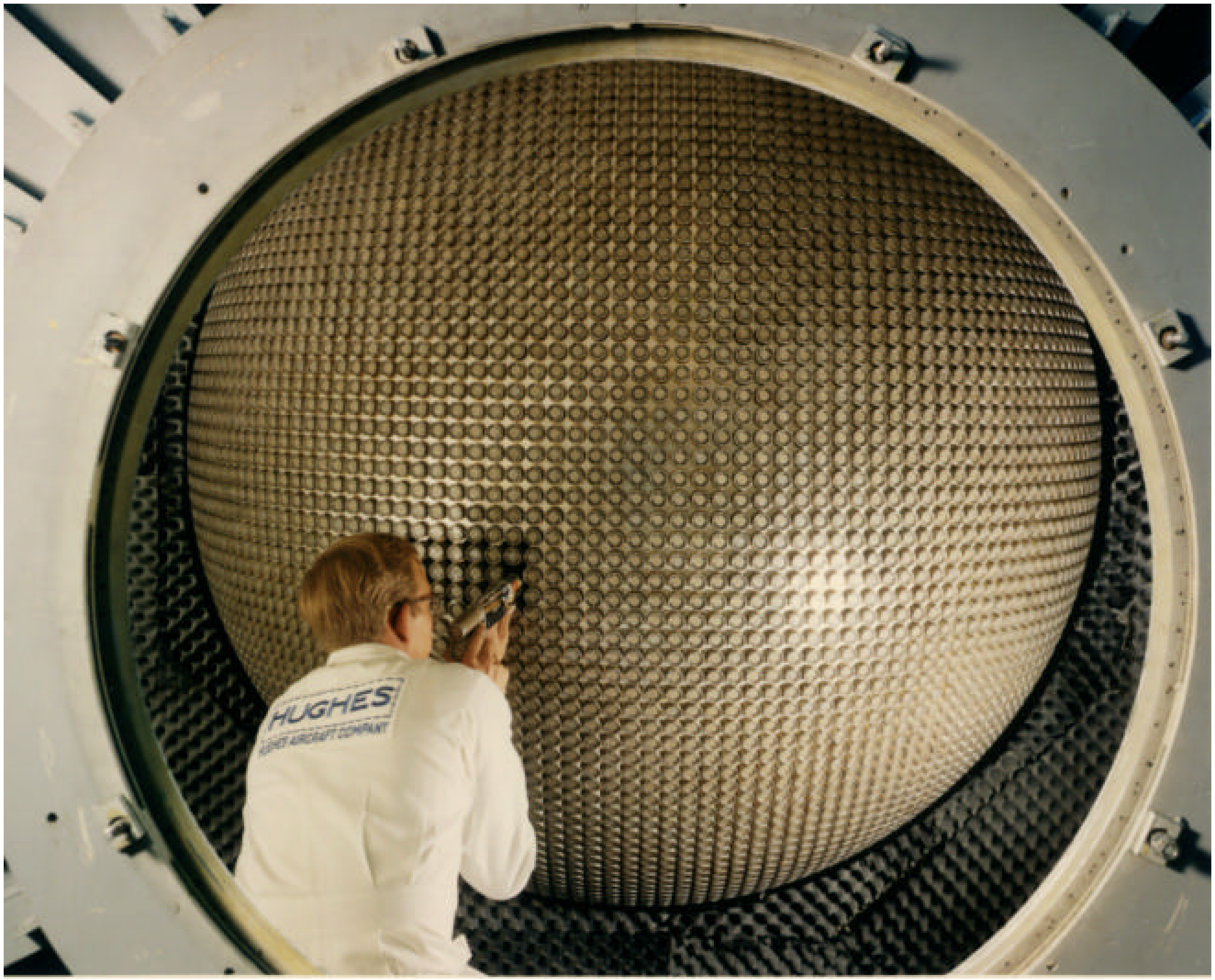
Reflectors and lenses can be designed with multiple focii. Surfaces more complicated that parabolas and hyperbolas are required, and often they are difficult to fabricate.

A Luneberg lens is a spherical structure that has a precisely controlled inhomogenous relative dielectric constant (or index of refraction, $n(r)$). Because of the spherical symmetry the feed can scan over 4π steradians. It is heavy and bulky.

$$n(r) = \sqrt{2 - (r/a)^2}$$



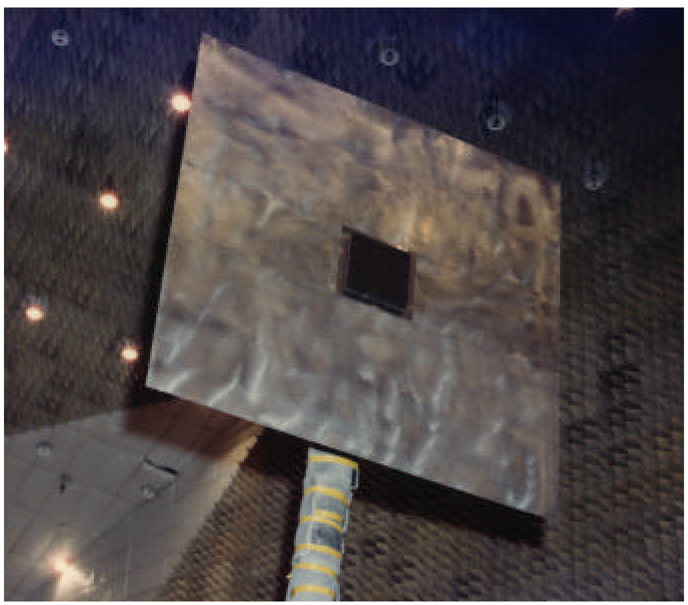
Lens Antenna



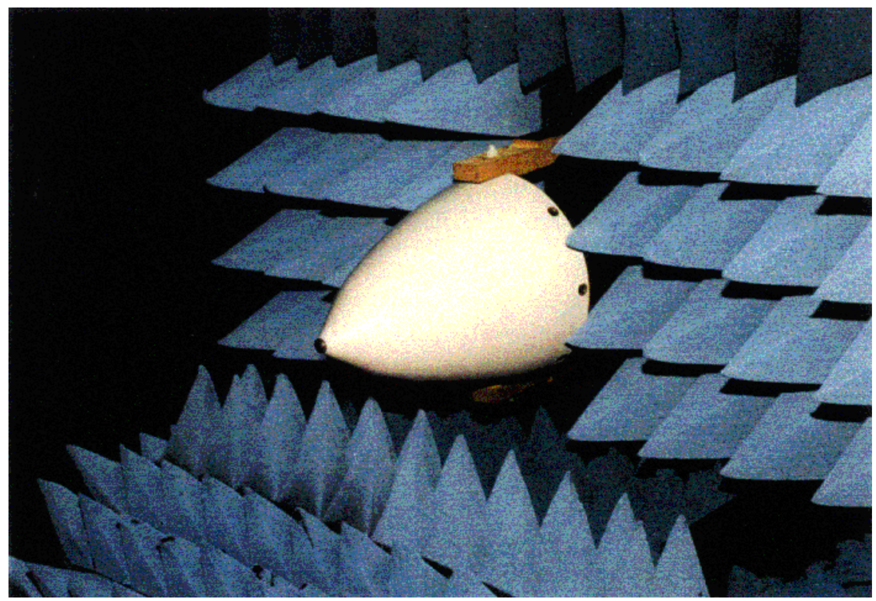
Radomes (1)

Radome, a term that originates from radar dome, refers to a structure that is used to protect the antenna from adverse environmental elements. It must be structurally strong yet transparent to electromagnetic waves in the frequency band of the antenna. Aircraft radomes are subjected to a severe operating environment. The heat generated by high velocities can cause ablation (a wearing away) of the radome material.

Testing of a charred space shuttle tile



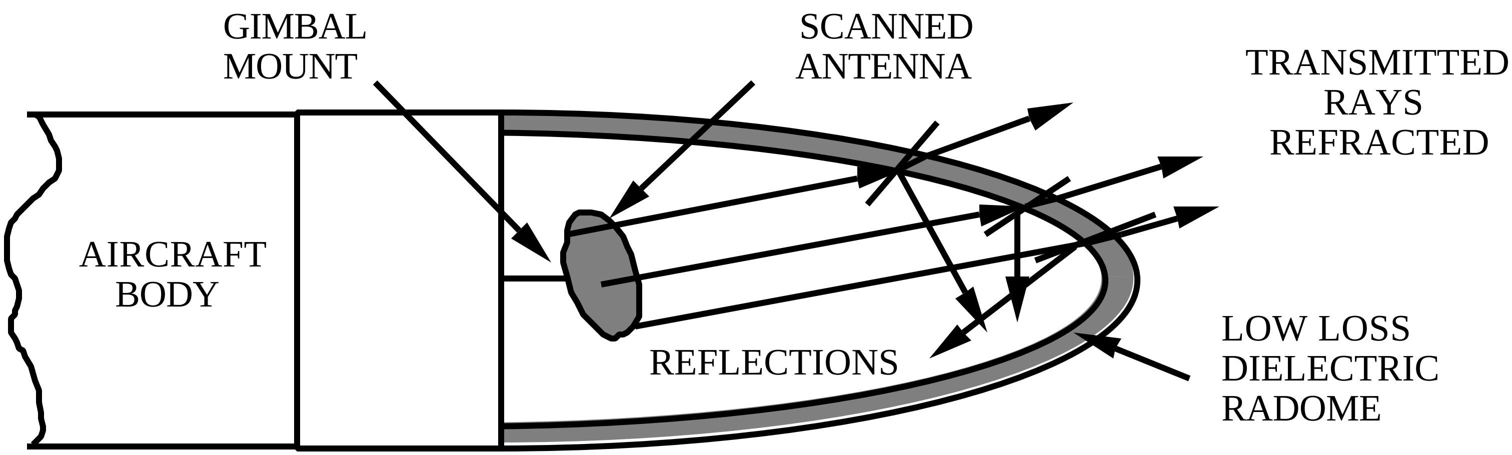
HARM (high-speed anti-radiation missile) radome testing



Radomes (2)

The antenna pattern with a radome will always be different than that without a radome. Undesirable effects include:

1. gain loss due to the dielectric loss in the radome material and multiple reflections
2. beam pointing error from refraction by the radome wall
3. increased sidelobe level from multiple reflections



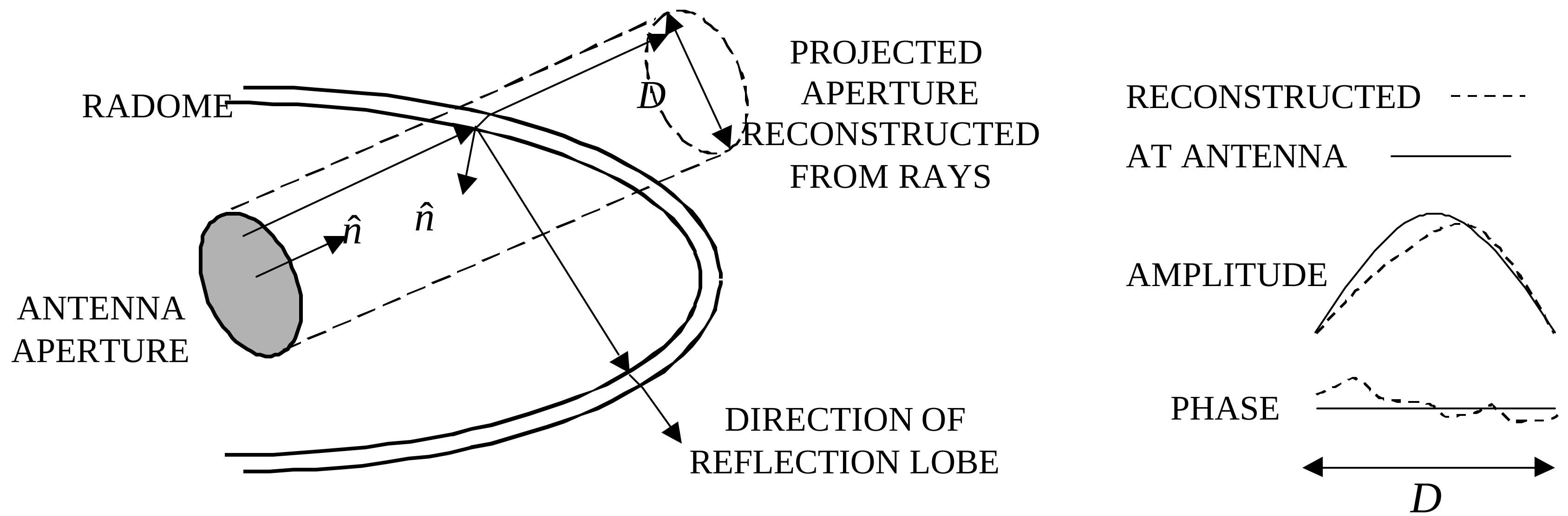
These effects range are small for flat non-scanning antennas with flat radomes, but can be severe for scanning antennas behind doubly curved radomes.

Radomes (3)

Geometrical optics can be used to estimate the effects of radomes on antenna patterns if the following conditions are satisfied:

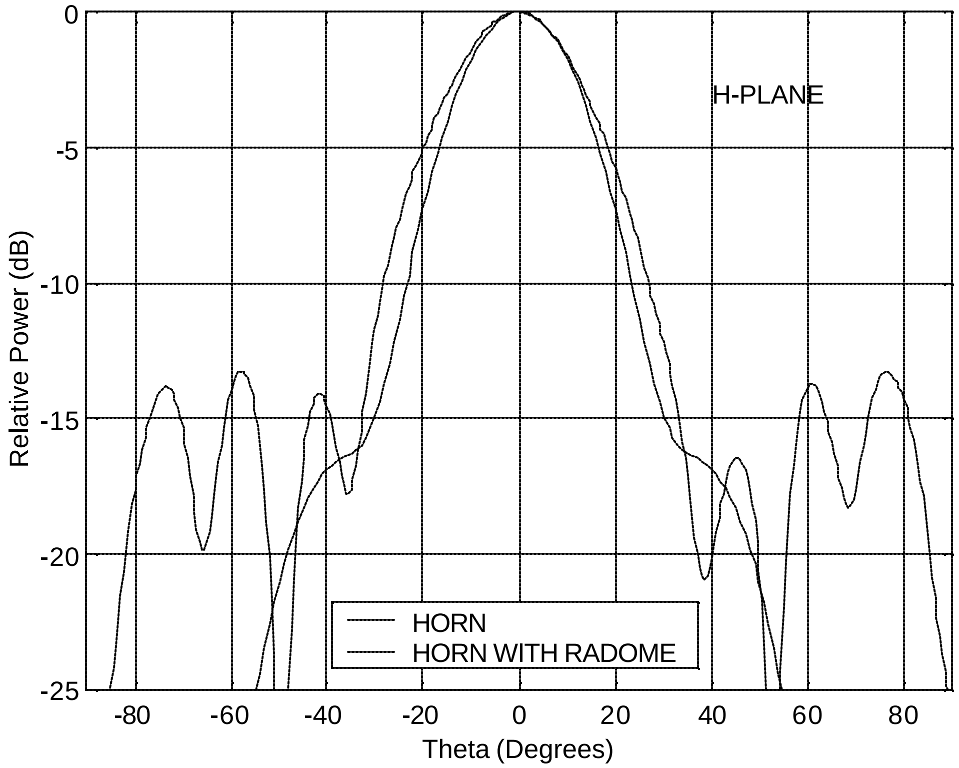
1. The radome is electrically large and its surfaces are "locally plane" (the radii of curvature of the radome surfaces are large compared to wavelength)
2. The radome is in the far field of the antenna
3. The number of reflections is small, so that the sum of the reflected rays converges quickly to an accurate result

Reconstructed aperture method:

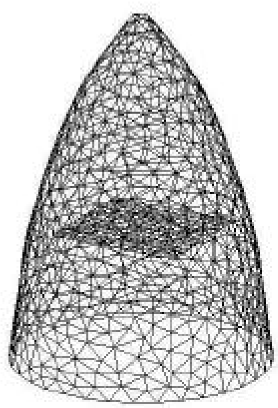


Radiation Pattern Effects of a Radome

Comparison of measured horn patterns
with and without a radome



Method of moments patch
model of a HARM radome



Hawkeye

JSTARS

Carrier Bridge

Antenna Measurements (1)

Purpose of antenna measurements:

1. Verify analytically predicted gain and patterns (design verification)
2. Diagnostic testing (troubleshooting)
3. Quality control (verify assembly methods and tolerances)
4. Investigate installation methods on patterns and gain
5. Determine isolation between antennas

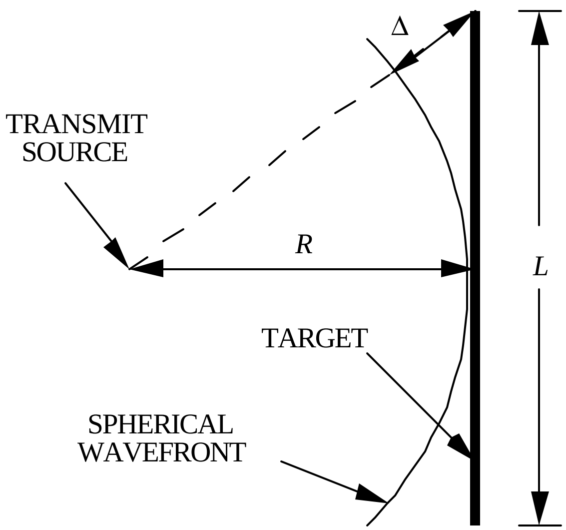
General measurement technique:

1. The measurement system is essentially a communication link with transmit and receive antennas separated by a distance R.
2. The antenna under test (AUT), that is, the antenna with unknown gain, is usually the receive antenna.
3. A calibration is performed by noting the received power level when a *standard gain horn* is used to receive (the gain of a standard gain horn is known precisely).
4. The AUT is substituted for the reference antenna, and the change in power is equivalent to the change in gain (since all other parameters in the Friis equation are the same for the two measurement conditions).

Antenna Measurements (2)

Conditions on the measurement facility include:

1. *R* must be large enough so that the spherical wave at the receive antenna is approximately a plane wave. (In other words, the receive antenna must be in the far field of the transmit antenna, and vice versa.)



The phase error at the edge of the antenna is typically limited to $p/8$

$$k\Delta_{\max} = k\sqrt{R^2 + (L/2)^2} - R = p/8$$

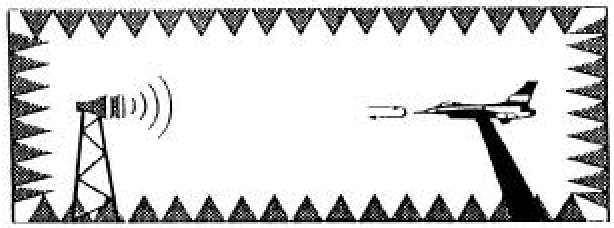
or,

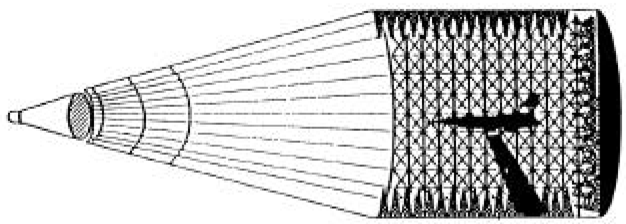
$$r_{ff} \equiv R_{\min} = \frac{2L^2}{l}$$

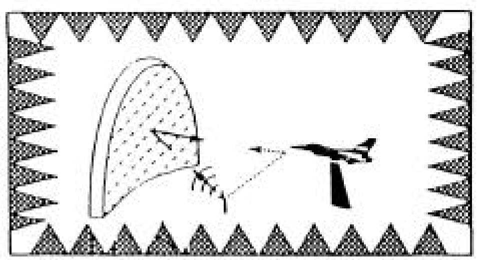
2. Reflections from the walls, ceiling and floor must be negligible so that multipath contributions are insignificant.
3. Noise in the instrumentation system must be low enough so that low sidelobe levels can be measured reliably.

Antenna Measurements (3)

Examples of measurement chambers. (AUTs are installed on an aircraft.)







Far field chamber: a communication link in a closed environment.

Tapered chamber: the tapered region behaves like a horn transition

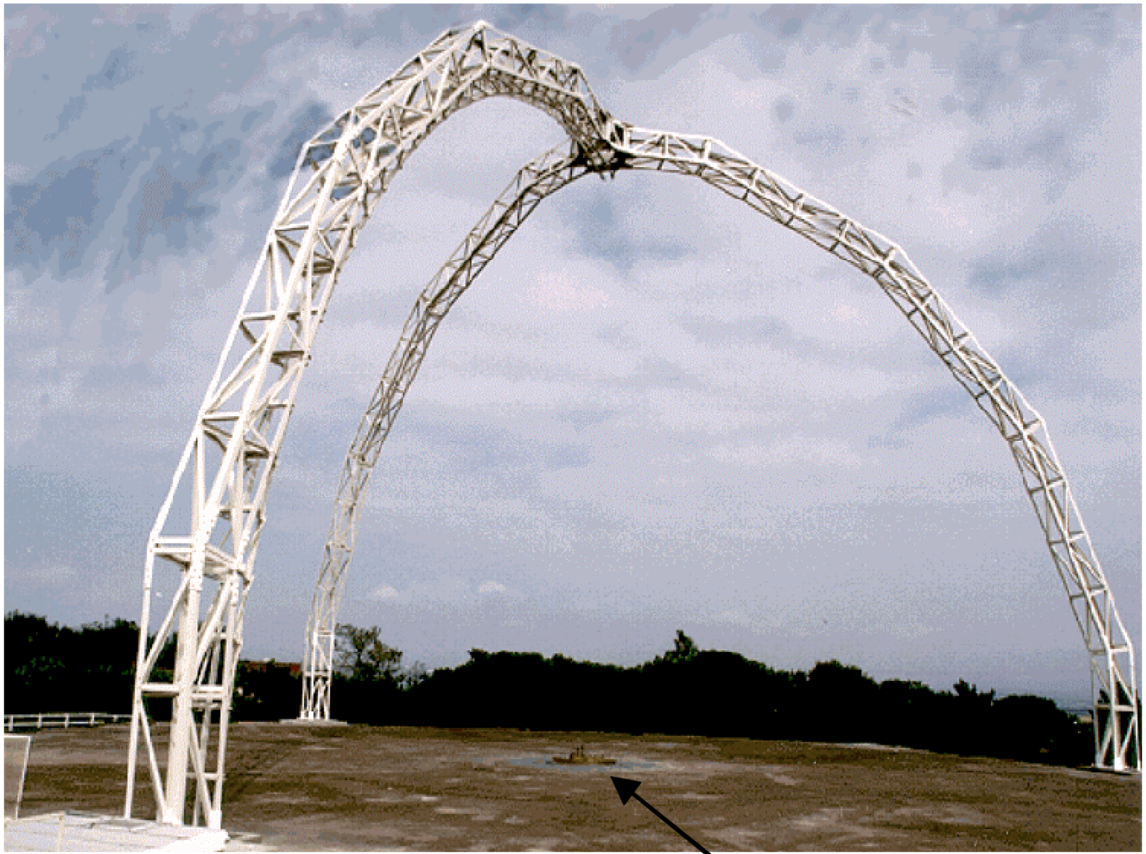
Compact range: a plane wave is reflected from the reflector, which allows very small values of R (mostly used for radar cross section and scattering measurements).

Antenna Measurements (4)

Antenna measurement facility descriptors:

SYSTEM DESCRIPTOR	CATEGORIES
physical configuration	indoor/outdoor
	near field
	far field
	compact
	tapered
instrumentation	time domain
	frequency domain
	continuous wave (CW)
	pulsed CW
data analysis & presentation	fixed frequency/variable aspect
	fixed aspect/frequency sweep
	two-dimensional frequency aspect
	time domain trace
	imaging of currents and fields
	polar or rectangular plots

NRAD Model Range at Point Loma



SHIP MODEL

Near-field Probe Pattern Measurement